

Chapter 13: Two Variables, One Line - Practice Question Bank

Easy Level

Multiple Choice Questions (MCQs)

1. Which of the following is a linear equation in two variables in standard form?
 - A) $x^2 + y = 5$
 - B) $3x + 4y - 12 = 0$
 - C) $xy + 2 = 0$
 - D) $3/x + y = 1$
2. In the linear equation $5x - 2y + 7 = 0$, what is the value of the constant term c ?
 - A) 5
 - B) -2
 - C) 7
 - D) 0
3. How many solutions does a linear equation in two variables have?
 - A) Exactly one
 - B) Exactly two
 - C) Infinitely many
 - D) Zero
4. The graph of every linear equation in two variables is always a:
 - A) Circle
 - B) Straight line
 - C) Parabola
 - D) Point
5. In the slope-intercept form $y = mx + d$, what does m represent?
 - A) Y-intercept
 - B) Slope
 - C) X-intercept
 - D) Constant

6. What is the slope of a horizontal line parallel to the x-axis?

- A) 1
- B) -1
- C) 0
- D) Undefined

7. What is the slope of a vertical line parallel to the y-axis?

- A) 0
- B) 1
- C) -1
- D) Undefined

8. If a line goes upward from left to right, its slope is:

- A) Positive
- B) Negative
- C) Zero
- D) Undefined

9. If a line goes downward from left to right, its slope is:

- A) Positive
- B) Negative
- C) Zero
- D) Undefined

10. Which ordered pair is a solution of the equation $2x + y = 7$?

- A) (1, 3)
- B) (2, 3)
- C) (0, 0)
- D) (3, 2)

11. If $c = 0$ in the standard form $ax + by + c = 0$, the line always passes through the:

- A) Origin (0, 0)
- B) Point (1, 1)
- C) Y-intercept at (0, 5)
- D) X-intercept at (5, 0)

12. The horizontal distance between two points on a line is called the:

- A) Rise
- B) Run
- C) Slope
- D) Intercept

13. The vertical distance between two points on a line is called the:

- A) Run
- B) Rise
- C) Slope
- D) Chord

14. What is the formula for the slope of a line passing through (x_1, y_1) and (x_2, y_2) ?

- A) $(x_2 - x_1) / (y_2 - y_1)$
- B) $(y_2 - y_1) / (x_2 - x_1)$
- C) $(x_1 + x_2) / 2$
- D) $y_2 - y_1 + x_2 - x_1$

15. When two lines in a system intersect at a single point, the system has:

- A) No solution
- B) Infinitely many solutions
- C) A unique solution
- D) Two solutions

16. When two lines in a system are parallel, the system has:

- A) A unique solution
- B) No solution
- C) Infinitely many solutions
- D) Two solutions

17. When two lines in a system are coincident, the system has:

- A) No solution
- B) A unique solution
- C) Infinitely many solutions
- D) Exactly two solutions

18. In the equation $y = -3x + 4$, what is the y-intercept d?

- A) -3

- B) 3
- C) 4
- D) -4

19. Which of the following represents a pair of linear equations in two variables?

- A) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
- B) $ax^2 + by + c = 0$
- C) $x + y + z = 0$
- D) $3x - 5 = 2$

20. What is the common method used to eliminate one variable by making coefficients equal?

- A) Substitution method
- B) Elimination method
- C) Graphical method
- D) Quadratic formula

Easy Level MCQ Answer Key

1. B
2. C
3. C
4. B
5. B
6. C
7. D
8. A
9. B
10. B
11. A
12. B
13. B
14. B
15. C
16. B
17. C
18. C
19. A
20. B

Easy Level Subjective Questions

1. Define a linear equation in two variables and state its standard form.

2. What does every point on the graph of a linear equation in two variables represent?
3. Explain the meaning of the terms 'rise' and 'run' in the context of the slope of a line.
4. What is the geometric meaning of a positive slope versus a negative slope?
5. State the condition for a linear equation in two variables to pass through the origin.
6. What is an ordered pair, and how is it used to denote a solution?
7. Explain what a slope-intercept form of a line represents.
8. How many solutions does a consistent and independent pair of linear equations have?
9. What does the intersection point of two lines represent in a graphical solution?
10. Describe the graphical appearance of two lines that have no solution.
11. Describe the graphical appearance of two lines that have infinitely many solutions.
12. Write down the slope of the line given by the equation $y = -5x + 3$.
13. Find two solutions for the linear equation $2x + y = 7$.
14. What happens to the slope calculation when dealing with a vertical line?
15. Explain why the equation $x = 3$ is considered a linear equation in two variables.
16. What are the two algebraic methods discussed for solving a pair of linear equations?
17. If a line has a slope of 0, describe its orientation on the Cartesian plane.
18. What is the coefficient of y in the standard form equation when rewritten as $y = mx + d$?
19. Give an example of a real-world situation that leads to a linear equation in two variables.
20. Why does the order of numbers matter in an ordered pair representing a solution?

Easy Level Subjective Answers

1. An equation of the form $ax + by + c = 0$, where a , b , and c are real numbers and a and b are not both zero, is called a linear equation in two variables.
2. Every point on the graph of a linear equation is a solution of that linear equation, and every solution corresponds to a point on the graph.
3. Run is the horizontal distance ($x_2 - x_1$) between two points, while rise is the vertical distance ($y_2 - y_1$) between them.
4. A positive slope means the line rises upward from left to right, whereas a negative slope means the line falls downward from left to right.
5. Whenever the constant term c is 0 in the standard form $ax + by + c = 0$, the line always passes through the origin $(0, 0)$.
6. An ordered pair is a pair of numbers written in a specific order, such as (x, y) , where the first number represents the x -value and the second represents the y -value.
7. The slope-intercept form is $y = mx + d$, where m represents the slope and d represents the y -intercept.
8. A consistent and independent pair of linear equations has a unique solution.
9. The intersection point of two lines represents the unique solution that satisfies both linear equations simultaneously.
10. Two lines that have no solution are strictly parallel and never intersect.
11. Two lines that have infinitely many solutions coincide completely and lie directly on top of each other.
12. The slope of the line $y = -5x + 3$ is -5 .
13. Two solutions for $2x + y = 7$ are $(0, 7)$ and $(3, 1)$.

14. For a vertical line, the run is zero, leading to division by zero, which makes the slope undefined.
 15. The equation $x = 3$ can be written in two variables as $1x + 0y - 3 = 0$.
 16. The two algebraic methods are the Substitution Method and the Elimination Method.
 17. A line with a slope of 0 is horizontal and parallel to the x-axis.
 18. The coefficient of y in the slope-intercept form when derived from $ax + by + c = 0$ is $m = -a/b$ (assuming $b \neq 0$).
 19. Buying mangoes and bananas for a fixed total cost, such as $60x + 50y = 280$.
 20. The order matters because the first number corresponds strictly to the coordinate along the horizontal axis (x) and the second to the vertical axis (y).
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Moderate Level

Multiple Choice Questions (MCQs)

1. Write the linear equation $5x + 2y = 2.7$ in standard form $ax + by + c = 0$. What is the value of c?
 - A) 2.7
 - B) -2.7
 - C) 5
 - D) 2
2. What is the slope of the line passing through the points A(-1, 0) and B(1, 2)?
 - A) 0
 - B) 1
 - C) 2
 - D) -1
3. If the point (-3, 4) is a solution of $3ax + 4y = -2$, what is the value of a?
 - A) 1
 - B) 2
 - C) -2
 - D) 3
4. What is the slope of the line represented by the equation $2x + 3y = 7$?
 - A) $2/3$
 - B) $-2/3$
 - C) $3/2$

- D) $-3/2$
5. For what condition does a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions?
- A) $a_1/a_2 \neq b_1/b_2$
 - B) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
 - C) $a_1/a_2 = b_1/b_2 = c_1/c_2$
 - D) $a_1/a_2 = c_1/c_2 \neq b_1/b_2$
6. For what condition does a pair of linear equations have no solution?
- A) $a_1/a_2 \neq b_1/b_2$
 - B) $a_1/a_2 = b_1/b_2 = c_1/c_2$
 - C) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
 - D) $a_1/a_2 \neq c_1/c_2$
7. Find the solution of the pair of equations $x + 2y = 3$ and $7x - 15y = 2$. What is the value of y ?
- A) $19/29$
 - B) $49/29$
 - C) $1/2$
 - D) $3/2$
8. What is the y-intercept of the line given by $3x - 7y = 21$?
- A) 3
 - B) -3
 - C) 7
 - D) -7
9. If two angles of a triangle are x degrees and y degrees with $y = 4x$, and the third angle is 50 degrees, what is the measure of x ?
- A) 26 degrees
 - B) 52 degrees
 - C) 104 degrees
 - D) 30 degrees
10. What is the slope of a line passing through the points (2, 0) and (4, 3)?
- A) $2/3$
 - B) $3/2$

- C) $\frac{1}{2}$
- D) 2

11. If the cost of 2 parathas and 7 bowls of dahi is 260, and 1 paratha and 2 bowls of dahi is 90, what is the cost of 1 bowl of dahi?

- A) 20
- B) 30
- C) 40
- D) 50

12. In the system $2mx + 3y = 7$ and $4x + ny = -10$, if $(2, -1)$ is a solution, what is the value of m ?

- A) 1
- B) 2
- C) 2.5
- D) 3

13. What is the slope of the line represented by $y = -5x + 3$?

- A) 3
- B) -3
- C) 5
- D) -5

14. Which of the following points lies on the line $3x + 2y = 12$?

- A) (1, 3)
- B) (2, 3)
- C) (4, 0)
- D) (0, 4)

15. If a line passes through $(-4, 5)$ and $(-1, 2)$, what is its slope?

- A) 1
- B) -1
- C) 3
- D) -3

16. What is the x-intercept of the line $2x + y = 7$?

- A) $\frac{7}{2}$
- B) 7

- C) 0
- D) $-7/2$

17. If a system of equations yields $0 = 0$ after subtraction during elimination, the system has:

- A) No solution
- B) Unique solution
- C) Infinitely many solutions
- D) Two solutions

18. What is the slope of the line passing through (4, 5) and taking run = 3, rise = 3?

- A) 0
- B) 1
- C) -1
- D) Undefined

19. A fraction becomes $9/11$ if 2 is added to both numerator and denominator. What is the equation formed by this condition?

- A) $9x - 11y = -4$
- B) $11x - 9y = 4$
- C) $9x + 11y = 2$
- D) $x + y = 20$

20. If the ratio of incomes of two persons is 9:7 and expenditures is 4:3, with savings 2000 each, what is the income of the first person?

- A) 14000
- B) 18000
- C) 20000
- D) 16000

Moderate Level MCQ Answer Key

1. B
2. B
3. B
4. B
5. C
6. C
7. A
8. B
9. A
10. B

11. A
12. C
13. D
14. B
15. B
16. A
17. C
18. B
19. A
20. B

Moderate Level Subjective Questions

1. Write the equation $x/2 - 4 = 3y/2$ in standard form and state the values of a, b, and c.
2. Find the slope of the line passing through the points A(-4, 5) and B(-1, 2) using the slope formula.
3. If (-3, 4) is a solution of $3ax + 4y = -2$ and $2x + by = 14$, find the values of a and b.
4. Verify whether the ordered pair (4, 3) is a solution of the linear equation $5x - 6y = 2$.
5. Find two solutions for the linear equation $7x - 3y = 21$, one on the x-axis and one on the y-axis.
6. In the equations $2mx + 3y = 7$ and $4x + ny = -10$, if (2, -1) is the solution of both, find the values of m and n.
7. Find two solutions that lie in different quadrants for the equation $5x + 3y = 7$ and verify them.
8. State whether the sentence 'A linear equation in two variables has only one solution' is True or False, and justify.
9. Explain how the slope-intercept form $y = mx + d$ can be derived from the standard form $ax + by + c = 0$.
10. Find the slope of a ramp at a loading dock that rises 2.5 metres over a horizontal run of 4 metres.
11. Construct a pair of linear equations for the following: On the first day, a family buys 2 movie tickets and 3 snack boxes for 850. On the second day, they buy 4 movie tickets and 1 snack box for 1100.
12. Solve the pair of equations $7x - 15y = 2$ and $x + 2y = 3$ using the substitution method.
13. The sum of a two-digit number and the number obtained by reversing its digits is 66. If the digits differ by 2, find the number.
14. Determine whether the lines represented by $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ intersect, coincide, or are parallel.
15. Form a pair of linear equations for: The sum of two integers is 5 and their difference is -21. Find the numbers.
16. The coach of a cricket team buys 7 bats and 6 balls for 8880. Later, she buys 3 bats and 5 balls for 4000. Form the pair of linear equations.
17. Use coefficient ratios to determine if the lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ intersect, are parallel, or coincide.

18. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Form the linear equations to find its dimensions.
19. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation such that the pair represents parallel lines.
20. Explain why multiplying a linear equation by a non-zero constant k results in an equivalent equation with the exact same set of solutions.

Moderate Level Subjective Answers

1. Rewriting $x/2 - 4 = 3y/2$ gives $x/2 - 3y/2 - 4 = 0$, with $a = 1/2$, $b = -3/2$, and $c = -4$ (or multiplying by 2 gives $x - 3y - 8 = 0$).
2. Slope = $(2 - 5) / (-1 - (-4)) = -3 / 3 = -1$.
3. Substituting $(-3, 4)$ gives $-9a + 16 = -2 \Rightarrow -9a = -18 \Rightarrow a = 2$. Substituting into the second gives $-6 + 4b = 14 \Rightarrow 4b = 20 \Rightarrow b = 5$.
4. Substituting $x = 4$ and $y = 3$ into $5x - 6y$ gives $5(4) - 6(3) = 20 - 18 = 2$, which matches the RHS. Thus, $(4, 3)$ is a solution.
5. Substituting $x = 0$ gives $y = -7$, so $(0, -7)$ is on the y-axis. Substituting $y = 0$ gives $x = 3$, so $(3, 0)$ is on the x-axis.
6. Substituting $(2, -1)$ into $2mx + 3y = 7$ gives $4m - 3 = 7 \Rightarrow 4m = 10 \Rightarrow m = 2.5$. Substituting into $4x + ny = -10$ gives $8 - n = -10 \Rightarrow n = 18$.
7. For $5x + 3y = 7$, substituting $x = 2$ gives $10 + 3y = 7 \Rightarrow y = -1$ (point $(2, -1)$ in Quadrant IV). Substituting $y = 4$ gives $5x + 12 = 7 \Rightarrow x = -1$ (point $(-1, 4)$ in Quadrant II).
8. False. A linear equation in two variables has infinitely many solutions because any assignment of value to x yields a valid corresponding value for y .
9. Starting with $ax + by + c = 0$, rearranging gives $by = -ax - c$, and dividing by b (assuming $b \neq 0$) gives $y = (-a/b)x + (-c/b)$, which matches $y = mx + d$ with $m = -a/b$ and $d = -c/b$.
10. Slope = Rise / Run = $2.5 / 4 = 0.625$.
11. Let ticket cost be x and snack box cost be y . Equations: $2x + 3y = 850$ and $4x + y = 1100$.
12. From $x + 2y = 3$, $x = 3 - 2y$. Substituting into $7x - 15y = 2$ gives $7(3 - 2y) - 15y = 2 \Rightarrow 21 - 14y - 15y = 2 \Rightarrow -29y = -19 \Rightarrow y = 19/29$, and $x = 49/29$.
13. Let digits be x and y . $x + y = 6$ and $x - y = 2$. Solving gives $x = 4$ and $y = 2$, so the number is 42.
14. Comparing coefficients: $a_1/a_2 = 1/2$, $b_1/b_2 = 2/4 = 1/2$, $c_1/c_2 = -4/-12 = 1/3$. Since $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, the lines are parallel and have no solution.
15. Let integers be x and y . Equations are $x + y = 5$ and $x - y = -21$.
16. Equations: $7x + 6y = 8880$ and $3x + 5y = 4000$ (where x is bat cost and y is ball cost).
17. Comparing $a_1/a_2 = 5/7$ and $b_1/b_2 = -4/6 = -2/3$. Since $a_1/a_2 \neq b_1/b_2$, the lines intersect at a single point.
18. Let width be y and length be x . $x = y + 4$ and $x + y = 36$.
19. Any equation with the same slope ratio but different intercept ratio, such as $2x + 3y - 12 = 0$.
20. Multiplying both sides of an equation by a non-zero constant k scales every term equally, preserving the equality relationship and the exact set of coordinate pairs that satisfy it.

Difficult Level

Multiple Choice Questions (MCQs)

1. If the lines represented by $kx + 2y = 3$ and $2x + 4y = 6$ are coincident, what is the value of k ?
 - A) 1
 - B) 2
 - C) 4
 - D) 3
2. For what value of p does the pair of equations $4x + py + 8 = 0$ and $x + 2y + 1 = 0$ have a unique solution?
 - A) $p = 8$
 - B) $p \neq 8$
 - C) $p = 2$
 - D) $p \neq 2$
3. Find the values of a and b for which the system $(a + b)x - 2by = 5a + 2b + 1$ and $3x - y = 14$ has infinitely many solutions. What is a ?
 - A) 1
 - B) 3
 - C) 5
 - D) 7
4. What is the value of k for which the system $x + 2y = 3$ and $(k - 1)x + (k + 1)y = k + 3$ represents coincident lines?
 - A) 2
 - B) 3
 - C) 4
 - D) 5
5. Robot 1 starts from origin and moves 3 right and 4 upward repeatedly. Robot 2 starts from $(10, 0)$ and moves 1 right and 2 upward repeatedly. Do their paths intersect?
 - A) No intersection
 - B) Intersect at $(3, 4)$
 - C) Intersect at $(6, 8)$

- D) Intersect at (10, 0)
6. The sum of the digits of a two-digit number is 15. The number obtained by interchanging digits exceeds the given number by 9. What is the number?
- A) 78
 - B) 87
 - C) 69
 - D) 96
7. In a cyclic quadrilateral ABCD, angle A = $(x + 7)$ degrees, angle B = $(y + 8)$ degrees, angle C = $(3y + 23)$ degrees, and angle D = $(4x + 12)$ degrees. What is the value of x?
- A) 10
 - B) 15
 - C) 20
 - D) 25
8. A train moving with uniform speed takes 6 hours less if speed is increased by 6 km/h, and 6 hours more if speed is decreased by 4 km/h. What is the speed of the train?
- A) 12 km/h
 - B) 24 km/h
 - C) 36 km/h
 - D) 48 km/h
9. The age of a father equals the sum of the ages of his four children. After 20 years, the sum of the children's ages will be twice the father's age. What is the father's current age?
- A) 30
 - B) 40
 - C) 50
 - D) 60
10. Plan A charges 50 monthly fee + 0.20 per minute. Plan B charges 30 monthly fee + 0.30 per minute. At what calling time do both plans cost the same?
- A) 100 minutes
 - B) 150 minutes
 - C) 200 minutes
 - D) 250 minutes
11. What is the x-intercept of the line $y = mx + c$?

- A) $-c/m$
- B) c/m
- C) $-m/c$
- D) c

12. If a system of equations has no solution, the graphs of the two equations are:

- A) Intersecting lines
- B) Coincident lines
- C) Parallel lines
- D) Perpendicular lines

13. How many lines exist that have a given slope and pass through a given point?

- A) 0
- B) 1
- C) 2
- D) Infinitely many

14. How many lines exist that have a given slope in the Cartesian plane?

- A) 1
- B) 2
- C) Infinitely many
- D) Zero

15. In Mahāvīrāchārya's problem from Gaṇita sāra saṅgraha, the price of 9 citrons and 7 wood-apples is 107, and 7 citrons and 9 wood-apples is 101. What is the sum of the price of one citron and one wood-apple?

- A) 11
- B) 13
- C) 15
- D) 21

16. What is the water level after 5 minutes if a water tank filling follows $y = 3x$?

- A) 10 cm
- B) 12 cm
- C) 15 cm
- D) 18 cm

17. How many minutes does it take for the water level to reach 21 cm in the tank following $y = 3x$?

- A) 5 minutes
- B) 6 minutes
- C) 7 minutes
- D) 8 minutes

18. In the temperature conversion formula $F = (9/5)C + 32$, at what temperature is the numerical value the same in both Fahrenheit and Celsius?

- A) -40
- B) 0
- C) 32
- D) -32

19. In a lending library with fixed charge for 3 days and additional charge per extra day, Saritha paid 27 for 7 days, and Susy paid 21 for 5 days. What is the fixed charge for 3 days?

- A) 9
- B) 12
- C) 15
- D) 18

20. A fraction becomes $5/6$ if 3 is added to both numerator and denominator. If 1 is added to numerator and 1 subtracted from denominator, fraction reduces to 1. What is the original fraction?

- A) $7/9$
- B) $5/7$
- C) $3/5$
- D) $2/3$

Difficult Level MCQ Answer Key

1. B
2. B
3. C
4. B
5. C
6. A
7. B
8. B
9. B

10. C
11. A
12. C
13. B
14. C
15. A
16. C
17. C
18. A
19. C
20. A

Difficult Level Subjective Questions

1. Find the value of k for which the pair of equations $kx + 2y = 3$ and $2x + 4y = 6$ has infinitely many solutions.
2. For what values of p does the pair of equations $4x + py + 8 = 0$ and $x + 2y + z = 0$ (or $x + 2y + 1 = 0$) have a unique solution?
3. Find the values of a and b for which the system of equations $(a + b)x - 2by = 5a + 2b + 1$ and $3x - y = 14$ has infinitely many solutions.
4. Find the value of k for which the system $x + 2y = 3$ and $(k - 1)x + (k + 1)y = k + 3$ represents a pair of coincident lines.
5. Robot 1 starts from origin and moves 3 right and 4 upward repeatedly. Robot 2 starts from $(10, 0)$ and moves 1 right and 2 upward repeatedly. Determine if their paths intersect and find the point of intersection.
6. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.
7. In a cyclic quadrilateral ABCD, angle $A = (x + 7)^\circ$, angle $B = (y + 8)^\circ$, angle $C = (3y + 23)^\circ$, and angle $D = (4x + 12)^\circ$. Find all four angles.
8. A train moving with uniform speed for a certain distance takes 6 hours less if its speed is increased by 6 km/h, and 6 hours more if its speed is decreased by 4 km/h. Find the distance and speed of the train.
9. The age of a father is equal to the sum of the ages of his four children. After 20 years, the sum of the children's ages will be twice the age of the father. Find the father's current age.
10. A person chooses between Plan A (50 monthly fee + 0.20/min) and Plan B (30 monthly fee + 0.30/min). Find the number of minutes at which both plans cost the same, and determine when Plan A is cheaper.
11. Give a formula to find the x -intercept of the line $y = mx + c$ and derive it.
12. Prove that two lines having the exact same slope must either be parallel or coincident.
13. Solve Mahāvīrāchārya's problem: 9 citrons and 7 wood-apples cost 107; 7 citrons and 9 wood-apples cost 101. Find the individual prices.
14. For the water tank graph $y = 3x$, determine if the point $(4, 10)$ lies on the line, and explain how to plot the intersection with $x = 3$.

15. Using the temperature conversion formula $F = (9/5)C + 32$, find the temperature that is numerically the same in both Fahrenheit and Celsius.
16. A lending library charges a fixed fee for the first 3 days and an extra daily fee thereafter. Saritha paid 27 for 7 days, and Susy paid 21 for 5 days. Find both charges.
17. A fraction reduces to 1 if 1 is added to numerator and 1 subtracted from denominator. It becomes $1/2$ if 1 is added only to the denominator. Find the fraction.
18. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. Find their present ages.
19. Meena withdrew 2000 from a bank using 50 and 100 notes only, receiving 25 notes in total. Find the number of notes of each denomination.
20. Solve the system of equations $2x + y = 6$ and $2x - y - 2 = 0$ graphically and algebraically, and find their intersection point.

Difficult Level Subjective Answers

1. For infinitely many solutions, $a_1/a_2 = b_1/b_2 = c_1/c_2 \Rightarrow k/2 = 2/4 = 3/6 \Rightarrow k/2 = 1/2 \Rightarrow k = 2$.
2. For a unique solution, $a_1/a_2 \neq b_1/b_2 \Rightarrow 4/1 \neq p/2 \Rightarrow p \neq 8$.
3. Equating coefficient ratios for infinitely many solutions gives $(a + b)/3 = -2b/-1 = (5a + 2b + 1)/14$. Solving these simultaneous relations yields $a = 5$ and $b = 3$.
4. For coincident lines, $1/(k - 1) = 2/(k + 1) = 3/(k + 3)$. Cross-multiplying gives $k + 1 = 2k - 2 \Rightarrow k = 3$.
5. Robot 1 path: $y = (4/3)x$. Robot 2 path: $y = 2(x - 10)$. Equating gives $(4/3)x = 2x - 20 \Rightarrow (2/3)x = -20 \Rightarrow x = -30$, $y = -40$. They intersect at $(-30, -40)$.
6. Let digits be x and y . $x + y = 15$ and $(10y + x) - (10x + y) = 9 \Rightarrow 9(y - x) = 9 \Rightarrow y - x = 1$. Solving gives $x = 7$, $y = 8$, so the number is 78.
7. Opposite angles of a cyclic quadrilateral are supplementary: $(x + 7) + (3y + 23) = 180$ and $(y + 8) + (4x + 12) = 180$. Solving the system yields $x = 15$ and $y = 38$. Angles are 22° , 46° , 137° , 72° .
8. Let distance be d and speed be s . Equations from time differences give $s = 24$ km/h and distance $d = 720$ km.
9. Let father's age be F and sum of children's ages be S . $F = S$. After 20 years, $S + 80 = 2(F + 20)$. Substituting F for S gives $F + 80 = 2F + 40 \Rightarrow F = 40$.
10. Equating costs: $50 + 0.20m = 30 + 0.30m \Rightarrow 0.10m = 20 \Rightarrow m = 200$ minutes. Plan A is cheaper for more than 200 minutes.
11. Set $y = 0$ in $y = mx + c \Rightarrow 0 = mx + c \Rightarrow mx = -c \Rightarrow x = -c/m$.
12. Two lines with slope m have equations $y = mx + d_1$ and $y = mx + d_2$. If $d_1 = d_2$, they are coincident; if $d_1 \neq d_2$, they have the same steepness and no intersection, making them parallel.
13. Adding equations: $16x + 16y = 208 \Rightarrow x + y = 13$. Subtracting equations: $2x - 2y = 6 \Rightarrow x - y = 3$. Solving gives $x = 8$, $y = 5$.
14. Substituting $x = 4$ into $y = 3x$ gives $y = 12 \neq 10$, so $(4, 10)$ does not lie on the line. At $x = 3$, $y = 3(3) = 9$, so intersection is at $(3, 9)$.
15. Set $F = C$ in $F = (9/5)C + 32 \Rightarrow C = (9/5)C + 32 \Rightarrow -(4/5)C = 32 \Rightarrow C = -40$.

16. Let fixed fee be x and daily fee be y . $x + 4y = 27$ and $x + 2y = 21$. Solving gives $x = 15$ (fixed fee) and $y = 3$ (daily fee).
17. Let fraction be x/y . $(x + 1)/(y - 1) = 1 \Rightarrow x - y = -2$. Also $x/(y + 1) = 1/2 \Rightarrow 2x - y = 1$. Solving gives $x = 3$, $y = 5$, so fraction is $3/5$.
18. Let Nuri be x and Sonu be y . $x - 5 = 3(y - 5) \Rightarrow x - 3y = -10$. Also $x + 10 = 2(y + 10) \Rightarrow x - 2y = 10$. Solving gives $x = 50$, $y = 20$.
19. Let 50 notes be x and 100 notes be y . $x + y = 25$ and $50x + 100y = 2000 \Rightarrow x + 2y = 40$. Solving gives $y = 15$ (100 notes) and $x = 10$ (50 notes).
20. For $2x + y = 6$ and $2x - y - 2 = 0$, adding gives $4x = 4 \Rightarrow x = 1$, and $y = 4$. Intersection point is $(1, 4)$.
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Olympiad Level

Multiple Choice Questions (MCQs)

- What is the area of the triangle formed by the lines $y = 3x$, $x = 3$, and the x -axis?
 - A) 9.5 sq units
 - B) 13.5 sq units
 - C) 18 sq units
 - D) 27 sq units
- If a system of three linear equations in two variables has a unique solution, what is the minimum number of independent equations required?
 - A) 1
 - B) 2
 - C) 3
 - D) 4
- For what values of k does the area of the triangle formed by the lines $x + y = 4$, axes, and $y = kx$ equal 8 square units?
 - A) $k = -1$ or $k = 3$
 - B) $k = 0$ or $k = 2$
 - C) $k = 1$ or $k = -3$
 - D) No such k exists
- If the vertices of a triangle are formed by the coordinate axes and the line $ax + by + c = 0$, what is the area of the triangle?
 - A) $c^2 / (2|ab|)$

- B) $|ab| / (2c^2)$
 - C) $c / (ab)$
 - D) $c^2 / (ab)$
5. What is the condition for three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, and $a_3x + b_3y + c_3 = 0$ to be concurrent (intersect at a single common point)?
- A) Determinant of coefficients equals zero
 - B) Sum of slopes equals zero
 - C) All intercepts are equal
 - D) Product of slopes is -1
6. If the distance between the two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $|c_1 - c_2| / \sqrt{a^2 + b^2}$, what is the distance between $3x + 4y - 12 = 0$ and $3x + 4y + 8 = 0$?
- A) 2
 - B) 4
 - C) 8
 - D) $20/7$
7. Jacob's digital watch reads 'a' minutes after 2 o'clock, and 15 minutes later reads 'b' minutes after 3 o'clock. If a is six times greater than b, what time was it when he looked the second time?
- A) 3:18
 - B) 3:21
 - C) 3:24
 - D) 3:30
8. What is the locus of a point whose distance from the x-axis is twice its distance from the y-axis?
- A) $y = 2x$
 - B) $y = 2|x|$
 - C) $2y = x$
 - D) $x^2 + y^2 = 4$
9. If the lines $2x + 3y = 5$, $5x + 7y = 12$, and $kx + 10y = 17$ are concurrent, what is the value of k?
- A) 6
 - B) 7
 - C) 8

- D) 9

10. What is the sum of the coordinates of the centroid of the triangle formed by the lines $x + y = 3$, $2x - y = 3$, and $x - 2y = 3$?

- A) 2
- B) 3
- C) 4
- D) 5

11. If a line passes through the intersection of $2x + y = 5$ and $x + 3y = 10$ and is parallel to the x-axis, what is its equation?

- A) $y = 3$
- B) $x = 2$
- C) $y = 5$
- D) $x = 3$

12. The number of integer solutions (x, y) satisfying $3x + 4y = 12$ with $x > 0$ and $y > 0$ is:

- A) 0
- B) 1
- C) 2
- D) Infinitely many

13. If the system $kx + y = 1$ and $x + ky = 1$ has infinitely many solutions, what is k ?

- A) 1
- B) -1
- C) 0
- D) No such real k exists

14. If the system $kx + y = 1$ and $x + ky = 1$ has no solution, what is k ?

- A) 1
- B) -1
- C) 0
- D) 2

15. What is the reflection of the point $(3, 4)$ across the line $y = x$?

- A) $(-3, -4)$
- B) $(4, 3)$
- C) $(-4, -3)$

- D) (3, -4)

16. If a line makes intercepts a and b on the coordinate axes, what is the equation of the line?

- A) $ax + by = 1$
- B) $x/a + y/b = 1$
- C) $x/b + y/a = 1$
- D) $y = mx + c$

17. What is the shortest distance from the origin to the line $3x + 4y - 12 = 0$?

- A) $12/5$
- B) $3/5$
- C) $4/5$
- D) 3

18. If the perimeter of a triangle formed by the coordinate axes and a line is 24 units, and the line passes through (3, 4), what is the equation of the line?

- A) $4x + 3y = 24$
- B) $3x + 4y = 24$
- C) $x + y = 7$
- D) $3x + 4y = 12$

19. If the system of equations $x + 2y = 3$, $2x + 3y = 4$, and $3x + 4y = k$ is consistent, what is the value of k ?

- A) 4
- B) 5
- C) 6
- D) 7

20. What is the angle between the lines represented by $y = m_1x + d_1$ and $y = m_2x + d_2$ when $m_1 * m_2 = -1$?

- A) 0 degrees
- B) 45 degrees
- C) 90 degrees
- D) 180 degrees

Olympiad Level MCQ Answer Key

1. B
2. B

3. A
4. A
5. A
6. C
7. B
8. B
9. B
10. A
11. A
12. A
13. A
14. B
15. B
16. B
17. A
18. B
19. B
20. C

Olympiad Level Subjective Questions

1. Find the area of the triangle bounded by the lines $y = 3x$, $x = 3$, and the x -axis.
2. Determine the condition for three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, and $a_3x + b_3y + c_3 = 0$ to be concurrent.
3. Solve Jacob's digital watch puzzle (Exercise end-of-chapter Q12): At a certain time, Jacob notices his digital watch reads 'a' minutes after 2 o'clock. Fifteen minutes later, it reads 'b' minutes after 3 o'clock. He notices a is six times greater than b. What time was it when he looked the second time?
4. Find the distance between the two parallel lines $3x + 4y - 12 = 0$ and $3x + 4y + 8 = 0$.
5. Find the value of k for which the lines $2x + 3y = 5$, $5x + 7y = 12$, and $kx + 10y = 17$ are concurrent.
6. Find the area of the triangle formed by the coordinate axes and the line $ax + by + c = 0$.
7. Prove that the area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$, and the y -axis is given by $(c_1 - c_2)^2 / (2|m_1 - m_2|)$.
8. Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y = 10$, and perpendicular to the line $x - 2y = 3$.
9. For what values of k does the system of equations $kx + y = 1$, $x + ky = 1$, and $x + y = k$ have a solution?
10. Find all integer solutions (x, y) satisfying the linear Diophantine equation $17x + 19y = 1000$.
11. Determine the coordinates of the orthocenter of the triangle formed by the lines $x + y = 4$, $2x - y = 2$, and $3x + 2y = 6$.
12. Prove that the family of lines given by $(2x + 3y - 4) + k(x - 2y + 1) = 0$ all pass through a fixed point, and find that point.

13. Find the locus of a point such that the sum of its distances from the coordinate axes is equal to 3.
14. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Find the speed of the stream and the boat in still water.
15. If the system of equations $ax + by = c$, $bx + cy = a$, and $cx + ay = b$ has non-trivial solutions, prove that $a^3 + b^3 + c^3 = 3abc$.
16. Find the shortest distance from the point (2, 3) to the line $4x + 3y - 10 = 0$.
17. Determine the condition for the lines $ax + by + c = 0$, $bx + cy + a = 0$, and $cx + ay + b = 0$ to form an equilateral triangle.
18. Find the coordinates of the circumcenter of the triangle formed by the lines $x = 0$, $y = 0$, and $x/a + y/b = 1$.
19. Prove that the point of intersection of the diagonals of any parallelogram formed by four lines $ax + by = c_1$, $ax + by = c_2$, $dx + ey = f_1$, and $dx + ey = f_2$ lies at the midpoint of its diagonals.
20. Solve the system of three linear equations in three variables: $x + y + z = 6$, $x - y + 2z = 5$, and $2x + y - z = 1$.

Olympiad Level Subjective Answers

1. The vertices of the triangle are (0, 0), (3, 0), and (3, 9). Area = $0.5 \times \text{base} \times \text{height} = 0.5 \times 3 \times 9 = 13.5$ square units.
2. Three lines are concurrent if the determinant formed by their coefficient matrix $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ equals zero.
3. Let first time be 2:a, so minutes passed from 2:00 is a. Second time is 3:(15 + b), so minutes passed from 2:00 is $60 + 15 + b = 75 + b$. Since 15 minutes elapsed, $75 + b = a + 15$. Given $a = 6b$, substituting gives $75 + b = 6b + 15 \Rightarrow 5b = 60 \Rightarrow b = 12$. Thus $a = 72$ (which means 3:12), and 15 minutes later is 3:27 (or wait, checking $75 + 12 = 87$ minutes from 2:00 = 3:27... let's verify: at first look it was 2:72 = 3:12, 15 mins later is 3:27).
4. Distance formula for parallel lines: $|c_2 - c_1| / \sqrt{a^2 + b^2} = |-12 - 8| / \sqrt{3^2 + 4^2} = 20 / 5 = 4$.
5. Solving the first two equations $2x + 3y = 5$ and $5x + 7y = 12$ gives intersection (1, 1). Substituting into $kx + 10y = 17$ gives $k(1) + 10(1) = 17 \Rightarrow k = 7$.
6. The intercepts on the axes are $x = -c/a$ and $y = -c/b$. Area = $0.5 \times |-c/a| \times |-c/b| = c^2 / (2|ab|)$.
7. Finding intersections of $y = m_1x + c_1$ and $y = m_2x + c_2$ with the y-axis gives heights and base length, yielding area = $(c_1 - c_2)^2 / (2|m_1 - m_2|)$ after integration or geometric derivation.
8. Intersection of $2x + y = 5$ and $x + 3y = 10$ is (1, 3). Line perpendicular to $x - 2y = 3$ has slope -2. Equation passing through (1, 3) with slope 0 (parallel to x-axis) or perpendicular: line perpendicular to $x - 2y = 3$ (slope 1/2) has slope -2, so $y - 3 = -2(x - 1) \Rightarrow 2x + y = 5$.
9. Using the determinant condition for consistency of three equations, $\det \begin{bmatrix} k & 1 & -1 \\ 1 & k & -1 \\ 1 & 1 & -k \end{bmatrix} = 0$ yields $k = 1$ or $k = -2$.

10. Using the Extended Euclidean Algorithm for $17x + 19y = 1000$, particular solution is found, and general solution is $x = 500 - 19k$, $y = -400 + 17k$ for integers k .
11. Finding the slopes of the sides and using altitude equations gives the orthocenter coordinates by solving simultaneous linear equations.
12. Rewriting as $(2x + 3y - 4) + k(x - 2y + 1) = 0$, setting $2x + 3y - 4 = 0$ and $x - 2y + 1 = 0$ yields fixed intersection point $(1, 1)$.
13. The locus consists of four line segments joining $(3,0)$, $(0,3)$, $(-3,0)$, and $(0,-3)$ forming a square rotated by 45 degrees.
14. Let upstream speed be u and downstream speed be v . $30/u + 44/v = 10$ and $40/u + 55/v = 13$. Solving gives $u = 5$ km/h and $v = 11$ km/h. Boat speed = 8 km/h, stream speed = 3 km/h.
15. For non-trivial solutions, the determinant of the coefficient matrix must be zero. Expanding $\det \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = 0$ leads directly to $a^3 + b^3 + c^3 = 3abc$.
16. Shortest distance formula: $|ax_0 + by_0 + c| / \sqrt{a^2 + b^2} = |4(2) + 3(3) - 10| / \sqrt{4^2 + 3^2} = |8 + 9 - 10| / 5 = 7/5$.
17. For an equilateral triangle, side lengths must be equal, which implies $a = b = c$ by symmetry of the cyclic coefficient arrangement.
18. The triangle is right-angled at origin $(0, 0)$. The circumcenter of a right-angled triangle is the midpoint of its hypotenuse, which is $(a/2, b/2)$.
19. The lines form a parallelogram with vertices at the intersections of the respective pairs. By coordinate geometry, the midpoint of the diagonal connecting opposite vertices is $((c_1+c_2)/2, (f_1+f_2)/2)$, proving coincidence.
20. Solving the 3x3 system: $x + y + z = 6$, $x - y + 2z = 5$, $2x + y - z = 1$ yields $x = 1$, $y = 2$, $z = 3$.