

Chapter 12: Quadrilaterals - Practice Question Bank

Easy Level

Multiple Choice Questions (MCQs)

1. What is a quadrilateral also commonly referred to as?
 - A) A 3-gon
 - B) A 4-gon
 - C) A 5-gon
 - D) A triangle
2. What is the sum of all internal angles of any convex quadrilateral?
 - A) 180 degrees
 - B) 270 degrees
 - C) 360 degrees
 - D) 540 degrees
3. A quadrilateral in which all four vertices do not lie in the same plane is called:
 - A) A convex quadrilateral
 - B) A non-convex quadrilateral
 - C) A non-planar quadrilateral
 - D) A self-intersecting quadrilateral
4. In a convex quadrilateral, all internal angles are:
 - A) Equal to 90 degrees
 - B) Less than 180 degrees
 - C) Greater than 180 degrees
 - D) Exactly 180 degrees
5. What is a quadrilateral with a 'dent' where one internal angle is greater than 180 degrees called?
 - A) Convex quadrilateral
 - B) Non-convex quadrilateral
 - C) Regular quadrilateral
 - D) Skew quadrilateral

6. How many diagonals does any planar quadrilateral have?

- A) 1
- B) 2
- C) 3
- D) 4

7. Which of the following is a quadrilateral whose opposite sides are parallel?

- A) Trapezium
- B) Parallelogram
- C) Triangle
- D) Pentagon

8. In a parallelogram, opposite sides are:

- A) Unequal and intersecting
- B) Equal and parallel
- C) Perpendicular
- D) Skew

9. In a parallelogram, opposite angles are:

- A) Supplementary
- B) Equal
- C) 90 degrees each
- D) Unrelated

10. The diagonals of a parallelogram always:

- A) Bisect each other
- B) Are perpendicular to each other
- C) Are of unequal length
- D) Do not intersect

11. What is the line segment joining the midpoints of two sides of a triangle called in relation to the third side?

- A) Perpendicular
- B) Parallel and has half the length
- C) Equal in length and perpendicular
- D) Double the length

12. What do we call the three line segments joining each vertex of a triangle to the midpoint of the opposite side?

- A) Altitudes
- B) Medians
- C) Angle bisectors
- D) Diagonals

13. The point where all three medians of a triangle intersect is called the:

- A) Orthocenter
- B) Circumcenter
- C) Centroid
- D) Incenter

14. In what ratio does the centroid divide each median of a triangle, starting from the vertex?

- A) 1:1
- B) 2:1
- C) 3:1
- D) 1:2

15. What quadrilateral is formed by joining the midpoints of the consecutive sides of any arbitrary quadrilateral (Varignon's Theorem)?

- A) Rectangle
- B) Square
- C) Parallelogram
- D) Rhombus

16. Tiling the plane means covering it with copies of a shape without:

- A) Gaps or overlaps
- B) Rotation
- C) Colors
- D) Touching edges

17. Which of the following shapes can be used to tile the plane?

- A) A regular pentagon only
- B) Any given quadrilateral (4-gon)
- C) No arbitrary quadrilaterals
- D) Only squares

18. What is the name of the mechanical linkage device based on parallelograms used to make exact scaled drawings?

- A) Lever
- B) Pantograph
- C) Pulley
- D) Compass

19. If the opposite sides of a quadrilateral are equal in length, what type of quadrilateral is it?

- A) Trapezium
- B) Parallelogram
- C) Kite
- D) Cyclic quadrilateral

20. What is the geometric name for a four-sided figure whose sides cross over each other like the figure CUTS?

- A) Convex quadrilateral
- B) Self-intersecting quadrilateral
- C) Non-planar quadrilateral
- D) Regular quadrilateral

Easy Level MCQ Answer Key

1. B
2. C
3. C
4. B
5. B
6. B
7. B
8. B
9. B
10. A
11. B
12. B
13. C
14. B
15. C
16. A
17. B
18. B
19. B
20. B

Easy Level Subjective Questions

1. Define a quadrilateral in your own words based on the text.
2. What is the difference between a convex quadrilateral and a non-convex quadrilateral?
3. State the three main properties of a parallelogram regarding its sides, angles, and diagonals.
4. State the converse of the property that the opposite sides of a parallelogram are equal.
5. State the Midpoint Theorem for triangles.
6. What is the converse of the Midpoint Theorem?
7. Define a median of a triangle.
8. State the Centroid Theorem regarding the medians of a triangle.
9. What is a Varignon parallelogram?
10. What does tiling the plane mean?
11. Give an example of a mechanical device that utilizes parallelograms for scaling drawings.
12. How many internal angles does a quadrilateral have, and what is their sum?
13. What condition distinguishes a planar quadrilateral from a non-planar (skew) quadrilateral?
14. What is a self-intersecting quadrilateral? Give an example from the text.
15. What are adjacent sides and opposite sides in a quadrilateral ABCD?
16. If the diagonals of a quadrilateral bisect each other, what can be concluded about the quadrilateral?
17. If a quadrilateral has one pair of opposite sides that are both equal and parallel, what is it?
18. What happens when you join the midpoints of the sides of any quadrilateral in order?
19. Why can any arbitrary quadrilateral tile the plane?
20. What is the ratio in which the centroid divides the medians of a triangle?

Easy Level Subjective Answers

1. A quadrilateral is a planar, non-self-intersecting geometric figure formed by four distinct line segments connecting four non-collinear points such that every point other than the vertices lies on exactly one side.
2. A convex quadrilateral has all internal angles strictly less than 180 degrees (and its diagonals intersect), whereas a non-convex quadrilateral has at least one internal angle greater than 180 degrees (a 'dent').
3. In a parallelogram: (a) opposite sides are equal, (b) opposite angles are equal, and (c) diagonals bisect each other.
4. If the opposite sides of a quadrilateral are equal in length, then it is a parallelogram.
5. The segment joining the midpoints of two sides of a triangle is parallel to the third side and has half its length.
6. The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.
7. A median is a line segment joining a vertex of a triangle to the midpoint of the opposite side.

8. The three medians of a triangle are concurrent at a point called the centroid, which divides each median in the ratio 2:1, with the longer segment connected to the vertex.
 9. The Varignon parallelogram is the parallelogram formed by connecting the midpoints of the four sides of an arbitrary quadrilateral in order.
 10. Tiling the plane means covering a flat plane completely with copies of a given shape or shapes without any gaps or overlaps.
 11. A pantograph is a mechanical device based on parallelograms used to make exact scaled copies of drawings.
 12. A quadrilateral has 4 internal angles, and their sum is always 360 degrees.
 13. All four vertices of a planar quadrilateral lie in the same flat plane, whereas a non-planar quadrilateral has vertices existing in three-dimensional space across different planes.
 14. A self-intersecting quadrilateral is a four-sided figure whose sides cross over each other (such as the figure CUTC).
 15. Adjacent sides share a common vertex endpoint, while opposite sides do not share any common vertex endpoint.
 16. If the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
 17. It is a parallelogram (equal parallel sides test).
 18. The resulting figure is always a parallelogram (Varignon's theorem).
 19. Any quadrilateral can tile the plane because copies can be rotated and fitted together such that their angles sum up to 360 degrees around vertices in repeating patterns.
 20. The centroid divides each median in a 2:1 ratio (measured from vertex to midpoint).
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Moderate Level

Multiple Choice Questions (MCQs)

1. If the diagonals of a parallelogram are equal, the parallelogram is necessarily a:
 - A) Rhombus
 - B) Rectangle
 - C) Square
 - D) Trapezium
2. If the diagonals of a parallelogram intersect at right angles (perpendicular), the parallelogram is a:
 - A) Rectangle
 - B) Rhombus
 - C) Trapezium
 - D) Kite only

3. What quadrilateral is formed by the pairwise intersection points of the four angle bisectors of any parallelogram where AB is not equal to BC ?
- A) Square
 - B) Rectangle
 - C) Rhombus
 - D) Trapezium
4. In $\triangle ABC$, if P and Q are midpoints of AB and AC , and $PQ = 4$ cm, what is the length of side BC ?
- A) 2 cm
 - B) 4 cm
 - C) 8 cm
 - D) 6 cm
5. If the midpoints of the sides of a rhombus are joined in order, the resulting quadrilateral is a:
- A) Rhombus
 - B) Rectangle
 - C) Square
 - D) Parallelogram
6. If the midpoints of the sides of a rectangle are joined in order, the resulting quadrilateral is a:
- A) Rectangle
 - B) Rhombus
 - C) Square
 - D) Trapezium
7. If the midpoints of the sides of a square are joined in order, the resulting quadrilateral is a:
- A) Rectangle
 - B) Rhombus
 - C) Square
 - D) Parallelogram
8. In $\triangle ABC$, the medians AP , BQ , and CR intersect at centroid M . If $CP = 9$ cm, what is the length of CM ?
- A) 3 cm

- B) 4.5 cm
 - C) 6 cm
 - D) 9 cm
9. In $\triangle ABC$, if M and N are midpoints of AB and AC, and D is any point on BC, what does segment MN do to segment AD?
- A) Trisects it
 - B) Bisects it
 - C) Is perpendicular to it
 - D) Equals its length
10. In a trapezium ABCD with AB parallel to DC and $AB \neq DC$, if G and H are the midpoints of diagonals AC and BD respectively, then GH is:
- A) Perpendicular to AB
 - B) Parallel to AB
 - C) Equal in length to AB
 - D) Zero length
11. What is the sum of internal angles of a self-intersecting quadrilateral where AB and CD intersect at E?
- A) Exactly 360 degrees
 - B) Less than 360 degrees
 - C) Greater than 360 degrees
 - D) Exactly 180 degrees
12. In a parallelogram ABCD, points P and Q are taken on diagonal BD such that $DP = BQ$. What quadrilateral is APCQ?
- A) Rectangle
 - B) Rhombus
 - C) Parallelogram
 - D) Square
13. If a triangle ABC is cut along its three midpoints into four smaller triangles, how are those four triangles related to each other?
- A) They are all congruent to each other
 - B) They are similar but not congruent
 - C) They have equal areas but different shapes
 - D) They are right-angled triangles

14. Which theorem is used directly to prove that the Varignon parallelogram of any quadrilateral is a parallelogram?

- A) Pythagorean Theorem
- B) Midpoint Theorem
- C) Centroid Theorem
- D) Angle Sum Property

15. If the diagonals of a quadrilateral bisect each other at right angles and are of equal length, the quadrilateral is a:

- A) Rectangle
- B) Rhombus
- C) Square
- D) Parallelogram

16. What is the auxiliary construction trick frequently used in proving the Midpoint Theorem?

- A) Drawing a circle through the vertices
- B) Drawing a line through one vertex parallel to another side
- C) Dropping a perpendicular from every vertex
- D) Bisecting all internal angles

17. If the opposite angles of a quadrilateral are equal, what can be deduced about its adjacent angles?

- A) They are equal
- B) They are supplementary (add up to 180 degrees)
- C) They are complementary
- D) They are 90 degrees each

18. In the tiling of a plane using an irregular quadrilateral SOME, how many copies of SOME fit around a common vertex point such that each angle 1, 2, 3, 4 is used once?

- A) 2 copies
- B) 3 copies
- C) 4 copies
- D) 6 copies

19. In a parallelogram, consecutive (adjacent) angles are always:

- A) Equal
- B) Supplementary
- C) Complementary

- D) Right angles

20. If a quadrilateral has diagonals that are perpendicular and bisect each other, it must be a:

- A) Rectangle
- B) Rhombus
- C) Square
- D) Trapezium

Moderate Level MCQ Answer Key

1. B
2. B
3. B
4. C
5. B
6. B
7. C
8. C
9. B
10. B
11. B
12. C
13. A
14. B
15. C
16. B
17. B
18. C
19. B
20. B

Moderate Level Subjective Questions

1. Explain why the converse of the property "opposite sides of a parallelogram are equal" is true and outline its proof using triangle congruence.
2. Explain how Theorem 3 (converse of opposite angles being equal) is proved using algebra and transversal properties.
3. Prove Theorem 4: A quadrilateral whose diagonals bisect each other is a parallelogram.
4. Explain the equal parallel sides test (Theorem 5) for a parallelogram.
5. Outline the proof of the Midpoint Theorem using the auxiliary line parallel to BA through C.
6. How does the second proof of Theorem 7 (Converse of the Midpoint Theorem) use the uniqueness of parallel lines?

7. Outline the geometric reasoning for the Centroid Theorem (Theorem 8) showing the 2:1 division ratio.
8. Prove Varignon's Theorem (Theorem 9): The midpoints of the four sides of any quadrilateral form a parallelogram.
9. In $\triangle ABC$, P and Q are midpoints of AB and AC. If BC = 12 cm, find the length of PQ and state the theorem used.
10. Let ABCD be a parallelogram where diagonal AC bisects angle A. Show that AC also bisects angle C and that ABCD is a rhombus.
11. Let ABCD be a parallelogram with $AB \neq BC$. Explain why the intersection points of the four angle bisectors form a rectangle.
12. In a quadrilateral ABCD, suppose $AB \parallel DC$ and $AB \neq DC$. If G and H are the midpoints of AC and BD, show that $GH \parallel AB$.
13. Prove that if the diagonals of a quadrilateral ABCD are equal and perpendicular, and Varignon's parallelogram is constructed, what special shape does the Varignon parallelogram take?
14. Explain how Lee Sallows' discovery in 2014 allows dividing a triangle into 3 congruent triangles after cutting along medians.
15. In a trapezium ABCD with $AB \parallel DC$, let E be the midpoint of AD and $EF \parallel AB$ meeting BC at F. Show that F is the midpoint of BC.
16. Let ABCD be a parallelogram and M, N be the midpoints of AB and CD. Show that segments DM and BN trisect diagonal AC.
17. Explain why the sum of angles of a self-intersecting quadrilateral is less than 360 degrees.
18. Describe Method 1 for tiling the plane with an arbitrary quadrilateral SOME using 180-degree rotations around edge midpoints.
19. Describe Method 2 for tiling the plane using the Varignon parallelogram grid of an arbitrary quadrilateral.
20. What is an aperiodic tiling, and what famous 2023 discovery exemplified this?

Moderate Level Subjective Answers

1. Let ABCD have $AB=CD$ and $AD=BC$. Triangle $ACD \cong \triangle CAB$ by SSS congruence. Corresponding angles are equal, so by transversal AC, $AB \parallel DC$ and $AD \parallel BC$, making it a parallelogram.
2. Given opposite angles are equal ($\angle A = \angle C$ and $\angle B = \angle D$), since the sum of angles in a quadrilateral is 360 degrees, $2(\angle A + \angle B) = 360^\circ$ implies $\angle A + \angle B = 180^\circ$. Since interior angles on the same side of transversal AB are supplementary, $AD \parallel BC$. Similarly, $AB \parallel DC$.
3. Let diagonals AC and BD intersect at E with $EA=EC$ and $EB=ED$. $\triangle AED \cong \triangle CEB$ by SAS congruence. Thus $\angle DAE = \angle ECB$, which by transversal AC implies $AD \parallel BC$. Similarly, $\triangle EAB \cong \triangle ECD$ implies $AB \parallel DC$. Thus ABCD is a parallelogram.

4. Given $AB \parallel DC$ and $AB=DC$. Let diagonals intersect at E. $\triangle EAB \cong \triangle ECD$ by ASA test, meaning $EA=EC$ and $EB=ED$ (diagonals bisect each other). By converse of property (c), ABCD is a parallelogram.
5. In $\triangle ABC$ with midpoints P and Q of AB and AC, draw line l through C parallel to BA intersecting line PQ at R. $\triangle APQ \cong \triangle CRQ$ by AAS, giving $PQ=QR$ and $CR=AP=BP$. Thus BCRP is a parallelogram, yielding $PQ \parallel BC$ and $PQ = BC/2$.
6. Draw line through midpoint P parallel to BC meeting AC at Q. Consider midpoint M of AC. By Midpoint Theorem, PM must be parallel to BC. Since there is a unique line through P parallel to BC, lines PM and PQ coincide, meaning Q must be the same point as M.
7. Let medians CP and BQ intersect at M. Let X and Y be midpoints of BM and CM. By Midpoint Theorem applied to $\triangle MBC$ and $\triangle ABC$, $XY \parallel BC$ and $PQ \parallel BC$. Congruence of triangles formed yields $CM = 2MP$ and $BM = 2MQ$, establishing the 2:1 ratio.
8. Let P, Q, R, S be midpoints of AB, BC, CD, DA. Applying Midpoint Theorem to $\triangle ABC$ gives $PQ \parallel AC$. Applying it to $\triangle ADC$ gives $SR \parallel AC$. Thus $PQ \parallel SR$. Similarly, $PS \parallel QR$. Therefore, PQRS is a parallelogram.
9. $PQ = \frac{1}{2}BC = \frac{1}{2}(12) = 6$ cm, by the Midpoint Theorem.
10. Since ABCD is a parallelogram, $AD \parallel BC$ and $AB \parallel CD$. If AC bisects $\angle A$, then $\angle DAC = \angle BAC$. Since $AD \parallel BC$, $\angle DAC = \angle BCA$ (alternate interior angles). Thus $\angle BAC = \angle BCA$, meaning $AB = BC$. Since opposite sides of a parallelogram are equal, all four sides are equal ($AB=BC=CD=DA$), making ABCD a rhombus.
11. The angle bisectors form a quadrilateral inside the parallelogram. Because adjacent angles of a parallelogram are supplementary ($\angle A + \angle B = 180^\circ$), half their sum is 90° . In the triangle formed by the bisectors at A and B, the third angle must be 90° . Repeating for all corners yields a rectangle.
12. Consider diagonal AC and BD. Using midpoint properties in triangles formed by the diagonals and sides AB and DC, segments connecting midpoints G and H are shown to be parallel to AB and equal to half the difference of AB and DC.
13. If the diagonals of a quadrilateral are equal and perpendicular, joining the midpoints of its sides (Varignon's parallelogram) results in a rhombus.
14. By cutting along medians into 6 triangles and reassembling adjacent pairs with matching equal sides, one obtains 3 congruent triangles with side lengths matching specific fractions of the original.
15. By drawing a diagonal of the trapezium (e.g., BD) and applying the Midpoint Theorem to the resulting triangles, the line EF intersecting BC at F forces F to be the midpoint of BC.
16. Let diagonals intersect. Using triangle similarity and midpoint properties (such as M and N being midpoints), segments DM and BN cut AC into three equal segments, trisecting it.

17. In a self-intersecting quadrilateral, the crossing of sides creates overlapping regions and inverted angle relationships, causing the sum of the interior angles to strictly fall below 360 degrees.
 18. Method 1 involves starting with two copies of SOME, rotating the top copy by 180 degrees around the midpoint of shared edge OM, and repeating this edge-to-edge rotation recursively to tile the plane.
 19. Method 2 involves constructing the grid of Varignon parallelograms for the given 4-gon SOME, placing copies of SOME matching the shaded parallelograms, and filling in the gaps automatically.
 20. An aperiodic tiling is a tiling of the plane by a set of shapes that tiles endlessly without ever forming a repeating periodic pattern. The 2023 discovery of the 'hat' shape by Smith, Myers, Kaplan, and Goodman-Strauss is a famous example.
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Difficult Level

Multiple Choice Questions (MCQs)

1. If the Varignon parallelogram of a quadrilateral is a rhombus, what must be true about the diagonals of the original quadrilateral?
 - A) They are equal in length
 - B) They are perpendicular to each other
 - C) They bisect each other at 60 degrees
 - D) They do not intersect
2. If the Varignon parallelogram of a quadrilateral is a rectangle, what must be true about the diagonals of the original quadrilateral?
 - A) They are perpendicular to each other
 - B) They are equal in length
 - C) They are parallel
 - D) They trisect each other
3. If the Varignon parallelogram of a quadrilateral is a square, what must be true about the diagonals of the original quadrilateral?
 - A) They are equal and perpendicular
 - B) They are unequal and parallel
 - C) They bisect angles but are unequal
 - D) They are skew

4. In a trapezium ABCD with $AD = 3$ cm and $BC = 5$ cm, and E, F as midpoints of non-parallel sides AB and CD, what is the length of segment EF?
- A) 3 cm
 - B) 4 cm
 - C) 5 cm
 - D) 8 cm
5. What fraction of the area of a triangle is bounded by the triangle formed by joining the midpoints of its three sides?
- A) $\frac{1}{2}$
 - B) $\frac{1}{3}$
 - C) $\frac{1}{4}$
 - D) $\frac{1}{8}$
6. In $\triangle ABC$, P is on AB such that $AP:PB = 1:2$, and PQ is parallel to BC meeting AC at Q. If $AQ = \sqrt{5}$, what is the length of QC?
- A) $\sqrt{5}$
 - B) $2\sqrt{5}$
 - C) $3\sqrt{5}$
 - D) 5
7. How many total quadrilaterals (including convex, non-convex, and self-intersecting) can be formed by 4 distinct points in a plane with no three collinear?
- A) 1
 - B) 2
 - C) 3
 - D) 4
8. Out of the total quadrilaterals formed by 4 non-collinear points, how many are convex?
- A) 1
 - B) 2
 - C) 3
 - D) 4
9. Out of the total quadrilaterals formed by 4 non-collinear points, how many are self-intersecting?
- A) 1
 - B) 2

- C) 3
- D) 4

10. What is the number of diagonals in an n -sided polygon (n -gon)?

- A) $\frac{n(n-3)}{2}$
- B) $\frac{n(n-1)}{2}$
- C) $n(n-3)$
- D) $\frac{n(n-2)}{2}$

11. What is the sum of internal angles of a planar convex n -gon?

- A) $(n-2) \times 180^\circ$
- B) $n \times 180^\circ$
- C) $(n-4) \times 180^\circ$
- D) 360°

12. In a non-planar quadrilateral ABCD, what is the sum of its internal angles?

- A) Exactly 360°
- B) Always less than 360°
- C) Always greater than 360°
- D) Exactly 180°

13. In a parallelogram ABCD, M and N are midpoints of AB and CD. Segments DM and BN are drawn. What do they do to diagonal AC?

- A) Bisect it
- B) Trisect it
- C) Quadrisect it
- D) Perpendicularly bisect it

14. If a quadrilateral has side lengths a, b, c, d arranged in increasing order, what condition must the largest side satisfy for the quadrilateral to exist?

- A) $d < a + b + c$
- B) $d > a + b + c$
- C) $d = a + b + c$
- D) $d \leq a + b + c$

15. In the general midpoint theorem for a trapezium where parallel sides are AB and CD , and EF passes through the midpoint of transversal AD parallel to AB , what is the length of EF ?

- A) $AB + CD$
- B) $\frac{AB + CD}{2}$
- C) $\frac{|AB - CD|}{2}$
- D) $\sqrt{AB \times CD}$

16. The discovery of aperiodic tilings using the 'hat' shape in 2023 was made by how many mathematicians?

- A) 2
- B) 3
- C) 4
- D) 5

17. If the diagonals of a parallelogram are equal and perpendicular, the parallelogram is a:

- A) Rectangle
- B) Rhombus
- C) Square
- D) Kite

18. In $\triangle ABC$, if the medians intersect at centroid M, what is the ratio of the area of $\triangle MAB$ to the total area of $\triangle ABC$?

- A) $\frac{1}{2}$
- B) $\frac{1}{3}$
- C) $\frac{1}{4}$
- D) $\frac{1}{6}$

19. When folding a right-triangle shaped paper cutout such that vertex A touches vertex B, the crease line represents the:

- A) Median to AB
- B) Perpendicular bisector of AB
- C) Angle bisector of C
- D) Altitude from C

20. Which famous mathematician/artist is well-known for tessellations and tilings featuring interlocking birds and animals?

- A) M.C. Escher
- B) Euclid
- C) Varignon
- D) Al-Khwarizmi

Difficult Level MCQ Answer Key

1. A
2. B
3. A
4. B
5. C
6. B
7. C
8. A
9. B
10. A
11. A
12. B
13. B
14. A
15. B
16. C
17. C
18. B
19. B
20. A

Difficult Level Subjective Questions

1. Prove that if the Varignon parallelogram of a quadrilateral is a rhombus, then the diagonals of the original quadrilateral are equal in length.
2. Prove that if the Varignon parallelogram is a rectangle, the diagonals of the original quadrilateral are perpendicular.
3. For a trapezium ABCD with parallel sides $AD = 3$ cm and $BC = 5$ cm, and E, F as midpoints of non-parallel sides, calculate the length of EF and justify using the trapezoidal midpoint theorem.
4. Derive the formula for the number of diagonals in an n -gon by analyzing how diagonals increase when adding a vertex.
5. Derive the formula for the sum of internal angles of an n -gon by triangulating from a single vertex.
6. Prove that the four smaller triangles formed by joining the midpoints of the sides of a triangle each have $\frac{1}{4}$ th the area of the original triangle.
7. Explain how segments DM and BN trisect the diagonal AC in a parallelogram ABCD where M and N are midpoints of AB and CD.
8. Discuss the existence conditions for a quadrilateral with given side lengths a, b, c, d .
9. Prove that the sum of internal angles of a non-planar quadrilateral is always strictly less than 360 degrees.
10. Explain the folding procedure in Exercise 5 where a right-triangle cutout is folded such that A touches B, and prove how the crease finds the midpoint of AC.

11. In $\triangle ABC$, if $AP:PB = 1:2$ and $PQ \parallel BC$, prove how to find QC given $AQ = \sqrt{5}$.
12. Prove that the medians of a triangle divide the triangle into six smaller triangles of equal area.
13. Analyze the properties of the Varignon parallelogram when the original quadrilateral is (i) a rectangle, (ii) a rhombus, (iii) a square.
14. Explain the geometric reasoning behind Theorem 9 (Varignon's Theorem) and identify the single exceptional case where the Varignon parallelogram collapses into a single line segment.
15. Detail the steps of Method 1 tiling using 180-degree rotations and explain why gaps do not form.
16. Detail the steps of Method 2 tiling using the Varignon parallelogram grid.
17. Explain the significance of aperiodic tilings in solid-state physics and the discovery of quasicrystals by Dan Shechtman.
18. Prove that the line segment joining the midpoints of the non-parallel sides of a trapezium is parallel to the parallel sides and equal to half their sum.
19. In a parallelogram $ABCD$, let O be the intersection of diagonals AC and BD . Prove that any line through O meeting AB and CD at P and Q is bisected at O ($PO = OQ$).
20. Discuss the concept of scissors congruence and cutting a triangle into 3 congruent triangles as discovered by Lee Sallows in 2014.

Difficult Level Subjective Answers

1. Using vector or coordinate geometry / triangle properties, the sides of the Varignon parallelogram are half the lengths of the diagonals of the original quadrilateral. If the Varignon parallelogram is a rhombus, its adjacent sides are equal, which implies the diagonals of the original quadrilateral are equal.
2. The sides of the Varignon parallelogram are parallel to the diagonals of the original quadrilateral. If the Varignon parallelogram is a rectangle, its adjacent sides are perpendicular, which means the diagonals of the original quadrilateral are perpendicular to each other.
3. By the trapezoidal midpoint theorem, the segment joining the midpoints of the non-parallel sides of a trapezium is equal to half the sum of the parallel bases: $EF = \frac{3 + 5}{2} = 4$ cm.
4. From any vertex of an n -gon, you can draw $n-3$ diagonals (since you cannot draw a diagonal to itself or its two adjacent vertices). Summing across all n vertices gives $n(n-3)$, but each diagonal is counted twice, yielding $\frac{n(n-3)}{2}$.
5. Choosing one vertex of an n -gon and drawing all non-intersecting diagonals from it divides the polygon into $n-2$ triangles. Since each triangle has an angle sum of 180° , the total angle sum is $(n-2) \times 180^\circ$.
6. Connecting the midpoints creates 4 triangles. By SAS congruence and the Midpoint Theorem, each smaller triangle is congruent to the others, and each has sides half the length of the original triangle, giving an area ratio of $(\frac{1}{2})^2 = \frac{1}{4}$.

7. In parallelogram ABCD with midpoints M and N, quadrilateral AMNC is a parallelogram, meaning $AM \parallel NC$ and $AM = NC$. Triangles formed with DM and BN establish similar triangle ratios that divide diagonal AC into three equal segments (trisecting it).
 8. By the generalized triangle inequality for polygons, the longest side must be strictly less than the sum of the remaining sides ($d < a + b + c$); otherwise, the sides cannot close to form a valid polygon.
 9. By treating a diagonal as a hinge and folding the non-planar quadrilateral into 3D space, the sum of projections of angles strictly decreases below 360° (reaching down to 2° in extreme folded states).
 10. Folding A to B creates the perpendicular bisector of segment AB as a crease line. By geometric properties and symmetry of the fold, this line also bisects segment AC (Midpoint Theorem / converse).
 11. Using the intercept theorem / proportionality (similar to the general form of the Midpoint Theorem in Grade 10), ratios of segments along AB match ratios along AC, allowing calculation of QC as $2\sqrt{5}$.
 12. The three medians divide the triangle into 6 smaller triangles. Because medians divide areas in specific proportions (centroid properties and equal base partitions), all 6 triangles have exactly equal area ($1/6$ of the total area).
 13. (i) Rectangle original $\implies$ Rhombus Varignon; (ii) Rhombus original $\implies$ Rectangle Varignon; (iii) Square original $\implies$ Square Varignon.
 14. Varignon's theorem relies on applying the Midpoint Theorem to triangles formed by the quadrilateral's diagonals. The exceptional case where the Varignon parallelogram collapses into a single segment occurs when the original quadrilateral is degenerate or self-intersecting in a specific collinear alignment.
 15. Method 1 uses 180-degree rotation around edge midpoints. Because opposite edges match in length and orientation under 180-degree rotation, they fit edge-to-edge without gaps or overlaps.
 16. Method 2 places copies of the 4-gon inside the grid formed by the Varignon parallelograms of the tiling, where vertex-to-vertex alignment automatically leaves gaps that are filled by remaining copies.
 17. Aperiodic tilings describe non-repeating ordered atomic structures found in quasicrystals, revolutionizing crystallography and solid-state physics (led to Dan Shechtman's 2011 Nobel Prize).
 18. Let trapezium ABCD have $AB \parallel CD$ with E and F as midpoints of AD and BC. Drawing a diagonal (e.g., BD) and applying the Midpoint Theorem to the resulting triangles shows $EF = \frac{AB + CD}{2}$ and $EF \parallel AB \parallel CD$.
 19. In parallelogram ABCD with diagonals intersecting at O, consider line through O meeting AB at P and QC at CD. $\triangle AOP \cong \triangle COQ$ by ASA congruence, which gives $PO = OQ$.
 20. Scissors congruence studies whether one polygon can be cut with straight cuts into pieces and reassembled into another. Lee Sallows discovered in 2014 how to cut a triangle and reassemble it into 3 congruent triangles.
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Olympiad Level

Multiple Choice Questions (MCQs)

1. What is the area of the Varignon parallelogram formed by joining the midpoints of the sides of a quadrilateral ABCD, in terms of the area of ABCD?
 - A) $\frac{\text{Area}(ABCD)}{4}$
 - B) $\frac{\text{Area}(ABCD)}{2}$
 - C) $\frac{\text{Area}(ABCD)}{3}$
 - D) $\text{Equal to Area}(ABCD)$
2. In a convex quadrilateral ABCD, if the area of triangle ABD is 12, triangle BCD is 15, triangle ABC is 14, what is the area of triangle ACD?
 - A) 13
 - B) 15
 - C) 16
 - D) 18
3. If a convex quadrilateral has perpendicular diagonals of lengths d_1 and d_2 , what is its area?
 - A) $d_1 d_2$
 - B) $\frac{1}{2} d_1 d_2$
 - C) $\frac{1}{4} d_1 d_2$
 - D) $2 d_1 d_2$
4. In a cyclic quadrilateral ABCD, if $\angle A = 70^\circ$, what is the measure of the opposite angle $\angle C$?
 - A) 70°
 - B) 90°
 - C) 110°
 - D) 180°
5. What is the maximum possible number of acute angles in a convex quadrilateral?
 - A) 1
 - B) 2
 - C) 3
 - D) 4

6. If the lengths of the sides of a cyclic quadrilateral are a, b, c, d , Brahmagupta's formula for its area uses $s = \frac{a+b+c+d}{2}$ as:
- A) $\sqrt{(s-a)(s-b)(s-c)(s-d)}$
 - B) $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$
 - C) $s(s-a)(s-b)(s-c)$
 - D) $\frac{1}{2}\sqrt{abcd}$
7. In any triangle ABC, what is the sum of the squares of the lengths of its three medians in terms of the sum of the squares of its side lengths a, b, c ?
- A) $\frac{3}{4}(a^2 + b^2 + c^2)$
 - B) $a^2 + b^2 + c^2$
 - C) $\frac{1}{2}(a^2 + b^2 + c^2)$
 - D) $\frac{4}{3}(a^2 + b^2 + c^2)$
8. Ptolemy's Theorem for a cyclic quadrilateral ABCD states that the product of the lengths of its diagonals is equal to:
- A) The sum of the opposite sides ($AB \cdot CD + BC \cdot DA$)
 - B) The product of opposite sides
 - C) The difference of opposite sides
 - D) The sum of adjacent sides
9. In $\triangle ABC$, if the centroid is G and P is any point in the plane, vectorially $\vec{PA} + \vec{PB} + \vec{PC}$ is equal to:
- A) $\vec{PG}$
 - B) $3\vec{PG}$
 - C) $\frac{1}{3}\vec{PG}$
 - D) Zero vector
10. What is the centroid of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$?
- A) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
 - B) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$
 - C) $(x_1+x_2+x_3, y_1+y_2+y_3)$
 - D) $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$
11. If a convex polygon has n sides, the sum of its exterior angles (one at each vertex, taken in order) is always:
- A) 180°
 - B) 360°

- C) $(n-2) \times 180^\circ$
- D) 540°

12. In a trapezium, the line segment joining the midpoints of the diagonals is equal to:

- A) $\frac{|AB + CD|}{2}$
- B) $\frac{|AB - CD|}{2}$
- C) $\frac{AB \cdot CD}{AB + CD}$
- D) Zero

13. Euler's line in a triangle passes through which of the following three notable points?

- A) Centroid, Orthocenter, Circumcenter
- B) Incenter, Centroid, Orthocenter
- C) Circumcenter, Incenter, Excenter
- D) Orthocenter, Incenter, Circumcenter

14. What is the ratio in which the centroid divides Euler's line between the orthocenter (H) and circumcenter (O)?

- A) 1:1
- B) 2:1 (with centroid twice as close to O as H)
- C) 1:2
- D) 3:1

15. If G is the centroid of $\triangle ABC$, what is the relationship between the areas of $\triangle GAB$, $\triangle GBC$, and $\triangle GCA$?

- A) They are in ratio 1:2:3
- B) They are all equal (each is $\frac{1}{3}$ of total area)
- C) They depend on side lengths
- D) Area of GAB is double the others

16. What is the minimum perimeter of a triangle inscribed in an acute-angled triangle ABC, known as Fagnano's problem?

- A) Formed by the feet of the altitudes (the orthic triangle)
- B) Formed by the medians
- C) Formed by the angle bisectors
- D) Formed by side midpoints

17. If a quadrilateral is inscribed in a circle of radius R and its diagonals are perpendicular, the sum of the squares of any two opposite sides is equal to:

- A) $2R^2$
- B) $4R^2$
- C) $8R^2$
- D) R^2

18. What is the area of a cyclic quadrilateral whose sides are a, b, c, d when it is also circumscribed about a circle (bicentric quadrilateral)?

- A) $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$ where $s = \frac{a+b+c+d}{2}$
- B) $\sqrt{abcd}$
- C) $\frac{1}{2}(ab+cd)$
- D) $s^2 - abcd$

19. In any convex quadrilateral, the sum of the lengths of the two diagonals is:

- A) Greater than the sum of any two opposite sides and less than the perimeter
- B) Equal to the perimeter
- C) Less than the sum of opposite sides
- D) Always equal to half the perimeter

20. The theorem stating that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of its diagonals is known as:

- A) Varignon's Theorem
- B) Parallelogram Law
- C) Ptolemy's Theorem
- D) Brahmagupta's Theorem

Olympiad Level MCQ Answer Key

1. B
2. A
3. B
4. C
5. C
6. B
7. A
8. A
9. B
10. A
11. B
12. B
13. A
14. B
15. B

16. A
17. B
18. B
19. A
20. B

Olympiad Level Subjective Questions

1. Prove that the area of the Varignon parallelogram of any quadrilateral is exactly half the area of the original quadrilateral.
2. State and prove Ptolemy's Theorem for cyclic quadrilaterals.
3. State and prove Brahmagupta's formula for the area of a cyclic quadrilateral.
4. Prove the Parallelogram Law: The sum of the squares of the lengths of the four sides of a parallelogram is equal to the sum of the squares of the lengths of its two diagonals ($2(a^2 + b^2) = d_1^2 + d_2^2$).
5. Prove that the sum of the squares of the lengths of the three medians of a triangle is equal to $\frac{3}{4}$ of the sum of the squares of the side lengths ($m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2)$).
6. Using vectors, prove that for any triangle ABC with centroid G and any point P in the plane, $\vec{PA} + \vec{PB} + \vec{PC} = 3\vec{PG}$.
7. Prove Euler's line theorem: In any non-equilateral triangle, the orthocenter (H), centroid (G), and circumcenter (O) are collinear, with G dividing segment HO in the ratio 2:1 ($HG:GO = 2:1$).
8. Prove that the line segment joining the midpoints of the diagonals of a trapezium has length equal to half the difference of the lengths of the parallel sides ($\frac{|AB - CD|}{2}$).
9. Prove that the area of a convex quadrilateral with diagonals d_1 and d_2 inclined at angle θ is given by $\text{Area} = \frac{1}{2} d_1 d_2 \sin \theta$.
10. Prove that a convex quadrilateral can be inscribed in a circle (is cyclic) if and only if its opposite angles are supplementary.
11. Prove that the orthic triangle of an acute-angled triangle (formed by the feet of its altitudes) has its sides bisected by the altitudes of the original triangle, and solve Fagnano's minimum perimeter problem.
12. Prove Newton's theorem on quadrilaterals: The midpoints of the two diagonals of a convex quadrilateral and the midpoint of the segment joining the intersections of opposite sides are collinear (the Newton line).
13. Analyze the properties of bicentric quadrilaterals (quadrilaterals that are both cyclic and circumscribed) and prove the relation for the distance between incenter and circumcenter (I and O): $\frac{1}{(R-d)^2} + \frac{1}{(R+d)^2} = \frac{1}{r^2}$.
14. Prove that the sum of the distances from any interior point of a triangle to its three vertices is less than the sum of its three side lengths (Erdős-Mordell inequality related property).
15. Investigate the properties of Simson's line in a triangle and how it relates to cyclic quadrilaterals formed by projection points on sides.

16. Prove that if a quadrilateral has perpendicular diagonals, the sum of the squares of two opposite sides equals the sum of the squares of the other two opposite sides ($AB^2 + CD^2 = BC^2 + DA^2$).
17. Using coordinate geometry or vector methods, prove that the midpoints of the four sides of any space quadrilateral (non-planar) still form a parallelogram.
18. Prove that the centroid of a triangle trisects the area, dividing $\triangle ABC$ into three triangles of equal area ($\text{Area}(GAB) = \text{Area}(GBC) = \text{Area}(GCA) = \frac{1}{3}\text{Area}(ABC)$).
19. Prove that in any convex quadrilateral, the sum of the lengths of the diagonals is greater than the semi-perimeter and less than the perimeter.
20. Explore the general case of Varignon's theorem in higher dimensions and non-Euclidean geometries, proving affine invariance of the Varignon parallelogram ratio.

Olympiad Level Subjective Answers

1. By dividing the quadrilateral into triangles along a diagonal and applying the Midpoint Theorem, each triangle's area associated with the Varignon parallelogram vertices equals half of that triangle's portion, summing to exactly half the total area of the quadrilateral.
2. Ptolemy's Theorem: For cyclic quadrilateral ABCD, $AC \cdot BD = AB \cdot CD + BC \cdot DA$. Proof involves constructing a point E on diagonal AC such that $\angle ABE = \angle DBC$, establishing similar triangles $\triangle ABE \sim \triangle DBC$ and $\triangle ABD \sim \triangle EBC$, and combining their ratio products.
3. Brahmagupta's Formula: Area of cyclic quadrilateral with sides a, b, c, d is $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$ where $s = \frac{a+b+c+d}{2}$. Proof uses law of cosines in triangles formed by diagonals and cyclic angle supplementary properties.
4. Parallelogram Law: $2(a^2 + b^2) = d_1^2 + d_2^2$. Proof uses the Law of Cosines on the acute and obtuse angles formed at the intersection of the diagonals or adjacent sides of the parallelogram.
5. Using Apollonius's Theorem (Stewart's theorem applied to medians), express each median in terms of sides: $m_a^2 = \frac{2b^2 + 2c^2 - a^2}{4}$. Summing $m_a^2 + m_b^2 + m_c^2$ yields $\frac{3}{4}(a^2 + b^2 + c^2)$.
6. Vector proof: Centroid G satisfies $\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$. For any point P, $\vec{PA} + \vec{PB} + \vec{PC} = (\vec{OA} - \vec{OP}) + (\vec{OB} - \vec{OP}) + (\vec{OC} - \vec{OP}) = 3\vec{OG} - 3\vec{OP} = 3(\vec{OG} - \vec{OP}) = 3\vec{PG}$.
7. Euler Line Proof: Using homothety (scaling by $-1/2$ centered at G) mapping the circumcenter O to the orthocenter H and mapping circumcircle to nine-point circle, establishing collinearity H-G-O with $HG:GO = 2:1$.
8. Let trapezium ABCD have $AB \parallel CD$ with diagonals AC and BD. Using midpoint properties of triangles formed by the diagonals, the segment joining the midpoints of AC and BD is parallel to AB and CD and has length $\frac{|AB - CD|}{2}$.
9. Area of convex quadrilateral with diagonals d_1, d_2 intersecting at angle θ is $\text{Area} = \frac{1}{2} d_1 d_2 \sin \theta$. Proof decomposes quadrilateral into four

triangles meeting at the diagonal intersection point O, summing their areas: $\frac{1}{2} AO \cdot DO \sin \theta + \dots = \frac{1}{2} d_1 d_2 \sin \theta$.

10. Cyclic Quadrilateral Theorem: Opposite angles sum to 180° ($\angle A + \angle C = 180^\circ$) iff ABCD is cyclic. Proof uses inscribed angle theorem and angle chasing around the circle.
11. Orthic Triangle and Fagnano's Problem: The altitudes of $\triangle ABC$ bisect the angles of its orthic triangle. By reflection principles across the sides of $\triangle ABC$, the perimeter is minimized when the inscribed triangle is the orthic triangle.
12. Newton's Line Theorem: The midpoints of the two diagonals of a convex quadrilateral and the midpoint of the segment joining the intersections of opposite extended sides are collinear. Proof uses mass points geometry or coordinate geometry.
13. Bicentric Quadrilateral Relation: For a bicentric quadrilateral with inradius r , circumradius R , and distance d between incenter and circumcenter, Euler's formula for bicentric quadrilaterals is $\frac{1}{(R-d)^2} + \frac{1}{(R+d)^2} = \frac{1}{r^2}$.
14. Erdős-Mordell Inequality / Geometric Property: The sum of distances from any interior point of a triangle to its vertices is at least twice the sum of distances to its sides ($PA + PB + PC \geq 2(PD + PE + PF)$).
15. Simson's Line Theorem: The projections of any point on the circumcircle of a triangle onto its three sides are collinear. The line containing these points is the Simson line.
16. Pythagorean Property of Perpendicular Diagonals: If diagonals are perpendicular, $AB^2 + CD^2 = BC^2 + DA^2$. Proof applies the Pythagorean theorem four times in the right-angled triangles formed around the diagonal intersection point O.
17. Space Quadrilateral Varignon Property: Even in 3D space (non-planar), vectors connecting the midpoints of adjacent sides satisfy parallel and equal conditions ($\vec{PQ} \parallel \vec{AC}$ and $\vec{SR} \parallel \vec{AC}$), ensuring the Varignon figure is still a parallelogram.
18. Centroid Area Trisecting Property: Medians divide $\triangle ABC$ into 6 smaller triangles of equal area. Since each of the three major triangles $\triangle GAB$, $\triangle GBC$, $\triangle GCA$ is composed of 2 of those 6 triangles, each has area equal to $1/3$ of the total area.
19. Diagonals Inequality in Convex Quadrilateral: The sum of lengths of diagonals is strictly greater than the sum of any two opposite sides and lies between the semi-perimeter and perimeter ($s < d_1 + d_2 < 2s$).
20. Affine Invariance of Varignon Parallelogram: Under any affine transformation (scaling, shearing, rotation), collinearity and midpoint ratios are preserved. Thus, the Varignon parallelogram property remains invariant across affine transformations.