

Chapter 9: Propositions and Their Converses - Complete Practice & Assessment Bank

EASY LEVEL

Easy Level: Multiple Choice Questions (MCQs)

1. Which of the following best defines a proposition?
 - A) A question that can have multiple answers
 - B) A statement that is either true or false
 - C) An algebraic expression with variables
 - D) A command given to solve a problem
2. The converse of the proposition "If X then Y" is:
 - A) If not X then not Y
 - B) If Y then X
 - C) If not Y then not X
 - D) X implies not Y
3. What is an example of a counterexample in mathematics?
 - A) An example that proves a theorem is universally true
 - B) An example that demonstrates the falsity of a given proposition
 - C) A duplicate formula used for verification
 - D) A step-by-step algebraic proof
4. Proposition: "If it rains, then the road is wet." What is its converse?
 - A) If it does not rain, then the road is not wet
 - B) If the road is wet, then it has rained
 - C) If the road is not wet, then it has not rained
 - D) If it rains, then the road is dry

5. Which of the following statements is true for any number n ?

- A) If n is a multiple of 6, then it is a multiple of 3
- B) If n is a multiple of 3, then it is a multiple of 6
- C) All multiples of 3 are multiples of 6
- D) If n is not a multiple of 3, it cannot be a multiple of 6

6. What is the converse of the statement: "If a number is a multiple of 6, then it is a multiple of 3"?

- A) If a number is not a multiple of 6, it is not a multiple of 3
- B) If a number is a multiple of 3, then it is a multiple of 6
- C) If a number is not a multiple of 3, then it is a multiple of 6
- D) If a number is a multiple of 3, then it is a multiple of 2

7. Is the converse of "If a number is a multiple of 6, then it is a multiple of 3" true?

- A) Yes, always
- B) No, because 3 is a multiple of 3 but not a multiple of 6
- C) Yes, because every number divisible by 3 is divisible by 6
- D) No, because 6 is not divisible by 3

8. Proposition: "If a number is a perfect square, then it has an odd number of factors." Which number serves as an example of a perfect square with an odd number of factors?

- A) 12
- B) 25
- C) 35
- D) 6

9. What is the converse of the Baudhayana-Pythagoras theorem statement: "If a triangle is right-angled, then $a^2 + b^2 = c^2$ "?

- A) If $a^2 + b^2 = c^2$, then the triangle is right-angled
- B) If $a^2 + b^2 \neq c^2$, then the triangle is acute
- C) If a triangle is equilateral, then $a^2 + b^2 = c^2$
- D) If $c^2 - a^2 = b^2$, then the triangle has an obtuse angle

10. Which phrase can convey the same meaning as "If X then Y"?

- A) X when Y
- B) Y when X
- C) X unless Y
- D) Y before X

11. What is the converse of: "If two lines are parallel, then the corresponding angles formed by a transversal are equal"?

- A) If corresponding angles are equal, then the lines are parallel
- B) If lines are not parallel, corresponding angles are equal
- C) If corresponding angles are unequal, lines are parallel
- D) If alternate interior angles are equal, lines are parallel

12. What is the converse of: "If a quadrilateral is a square, then all its angles are equal"?

- A) If all angles of a quadrilateral are equal, then it is a square
- B) If a quadrilateral is a rectangle, it is a square
- C) If sides are equal, it is a square
- D) If angles are unequal, it is a rhombus

13. Given x and y are real numbers, what is the converse of: "If x equals y, then $x^2 = y^2$ "?

- A) If $x^2 = y^2$, then x equals y
- B) If $x^2 \neq y^2$, then x does not equal y

- C) If x does not equal y , then x squared does not equal y squared
 - D) If x equals negative y , then x squared equals y squared
14. What is the converse of: "If n is divisible by 24, then it is divisible by both 4 and 6"?
- A) If n is divisible by 4 and 6, then it is divisible by 24
 - B) If n is divisible by 24, it is divisible by 12
 - C) If n is not divisible by 24, it is not divisible by 4
 - D) If n is divisible by 3, it is divisible by 24
15. Is the statement "If n is divisible by 4 and 6, then it is divisible by 24" always true?
- A) Yes, because 4 times 6 equals 24
 - B) No, because 12 is divisible by 4 and 6, but not by 24
 - C) Yes, all common multiples of 4 and 6 are multiples of 24
 - D) No, because 24 has no factors other than itself
16. What is the converse of: "If n is the square of a prime number, then it has exactly 3 factors"?
- A) If n has exactly 3 factors, then it is the square of a prime number
 - B) If n has 4 factors, it is a prime number
 - C) If n is prime, it has 3 factors
 - D) If n is not a square, it has odd factors
17. Express the relationship using an 'If-then' sentence: "A number is divisible by 3 when the sum of its digits is a multiple of 3."
- A) If a number is divisible by 3, then the sum of its digits is a multiple of 3
 - B) If the sum of the digits is a multiple of 3, then the number is divisible by 3
 - C) If a number is not divisible by 3, the sum of digits is zero
 - D) If a number is a multiple of 3, it is even

18. In the context of propositions, a counterexample is used to show that:

- A) A proposition is true
- B) A proposition is false
- C) A proposition has a converse
- D) Two propositions are identical

19. Which of the following statements is a proposition?

- A) Wow, what a triangle!
- B) Close the door.
- C) The sum of angles of a triangle is 180 degrees.
- D) Where is the centre of the circle?

20. What can happen to a proposition and its converse?

- A) Both can be true
- B) Both can be false
- C) One can be true and the other false
- D) All of the above

Easy Level: MCQ Answer Key

1. B | 2. B | 3. B | 4. B | 5. A | 6. B | 7. B | 8. B | 9. A | 10. B | 11. A | 12. A | 13. A | 14. A | 15. B | 16. A | 17. B | 18. B | 19. C | 20. D

Easy Level: Subjective Questions

1. Define what a proposition is in mathematics. Give one example.
2. What is meant by the converse of a proposition of the form 'if X then Y'?
3. Explain the term 'counterexample' in your own words and state why it is important.
4. State the converse of the proposition: "If it rains, then the road is wet." Is the converse necessarily true? Give a brief reason.
5. Consider the proposition: "If a number is a multiple of 6, then it is a multiple of 3." State its converse.
6. Give a counterexample to show that the converse of "If a number is a multiple of 3, then it is a multiple of 6" is false.

7. State whether the proposition "If n is a perfect square, then it has an odd number of factors" is true or false.
8. What is the converse of the Baudhayana-Pythagoras theorem?
9. Write the sentence "A number has an odd number of factors when it is a perfect square" in an equivalent 'If-then' form.
10. Frame the converse of the proposition: "If two lines are parallel, then the corresponding angles formed by a transversal are equal."
11. Frame the converse of the proposition: "If a quadrilateral is a square, then all its angles are equal."
12. Frame the converse of the proposition: "If x equals y , then $a + x$ equals $a + y$."
13. Frame the converse of the proposition: "If a and b are perfect squares, then $a \times b$ is a perfect square."
14. Frame the converse of the proposition: "If x equals y , then x^3 equals y^3 " (where x and y are real numbers).
15. Frame the converse of the proposition: "If n is divisible by 60, then it is divisible by both 5 and 12."
16. Frame the converse of the proposition: "If n is the square of a prime number, then it has exactly 3 factors."
17. Write the divisibility rule shortcut for checking whether a number is divisible by 3 in the form of an 'If-then' proposition.
18. Give an everyday real-life example of a statement where the proposition is true, but its converse is false.
19. True or False: If a proposition is true, its converse must also always be true. Justify with a simple comment.
20. Complete the sentence from the chapter summary: "Counterexamples are instances that demonstrate the ____ of a given proposition."

Easy Level: Subjective Answers

1. A proposition is a statement that is either true or false. Example: "A triangle has three sides."
2. The converse of 'if X then Y ' is formed by reversing the clauses to get 'if Y then X '.
3. A counterexample is an example that contradicts a stated proposition; it is important because it allows us to show quickly and convincingly that a proposition is false.

4. Converse: "If the road is wet, then it has rained." It is not necessarily true because the road could be wet due to a water tanker spill.
 5. Converse: "If a number is a multiple of 3, then it is a multiple of 6."
 6. The number 3 is a multiple of 3, but it is not a multiple of 6 (or 9, 15, etc.).
 7. True.
 8. Converse: "Let a , b , c be the sidelengths of a triangle. If $a^2 + b^2 = c^2$, then the triangle is right-angled."
 9. "If a number is a perfect square, then it has an odd number of factors."
 10. Converse: "If the corresponding angles formed by a transversal are equal, then two lines are parallel."
 11. Converse: "If all angles of a quadrilateral are equal, then it is a square."
 12. Converse: "If $a + x = a + y$, then $x = y$."
 13. Converse: "If $a \times b$ is a perfect square, then a and b are perfect squares."
 14. Converse: "If $x^3 = y^3$, then $x = y$."
 15. Converse: "If n is divisible by both 5 and 12, then it is divisible by 60."
 16. Converse: "If n has exactly 3 factors, then it is the square of a prime number."
 17. "If the sum of the digits of a number is a multiple of 3, then the number is divisible by 3."
 18. "If a person lives in Paris, then they live in France." (True, but converse "If a person lives in France, they live in Paris" is false).
 19. False. A proposition can be true while its converse is false.
 20. Falsity.
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MODERATE LEVEL

Moderate Level: Multiple Choice Questions (MCQs)

1. Who disproved Fermat's claim that all numbers of the form $2^n + 1$ are prime by showing the case for $n = 5$ is composite?
 - A) Pythagoras
 - B) Euler

- C) Baudhayana
 - D) Euclid
2. What is the value of 2 to the power of (2 to the power of 3) plus 1 as mentioned in the text?
- A) 17
 - B) 257
 - C) 65537
 - D) 9
3. Consider the proposition: "If a number is a multiple of 6, then it is a multiple of 3." Why is this proposition true?
- A) Because 6 is a multiple of 3, any multiple of 6 is also a multiple of 3.
 - B) Because 3 is a factor of 6.
 - C) Both A and B.
 - D) It is actually false.
4. Which of the following statements about the factors of numbers is correct?
- A) Every number has an odd number of factors.
 - B) Only prime numbers have factor pairs.
 - C) Each factor of a number has a 'partner' factor such that their product yields the given number.
 - D) Perfect squares always have an even number of factor pairs with repeating factors.
5. Why does a perfect square have an odd number of factors?
- A) Because one factor-partner pair consists of the same number repeating (f times f).
 - B) Because primes have 2 factors.
 - C) Because square numbers are always divisible by 4.
 - D) Because factor pairs never repeat.

6. What can be said about the number of factor pairs with repeating factors (f, f) in a square number?

- A) A square number can have more than two repeating pairs.
- B) A square number cannot have more than one factor pair with repeating factors.
- C) A square number has zero repeating pairs.
- D) A square number has repeating pairs equal to its square root.

7. Proposition: "If two triangles have the same area, then they are congruent." Is this proposition true?

- A) Yes, area determines congruence.
- B) No, triangles with different shapes and sides can have the same area.
- C) Yes, by SAS criterion.
- D) Only for right-angled triangles.

8. Converse: "If two triangles are congruent, then they have the same area." Is this statement true?

- A) No, congruent triangles have different areas.
- B) Yes, congruent shapes have identical areas.
- C) False, because congruence only applies to angles.
- D) Only true for equilateral triangles.

9. What geometric criterion is used in the text to prove the converse of the Baudhayana-Pythagoras theorem?

- A) SAS criterion
- B) ASA criterion
- C) SSS criterion for congruence
- D) RHS criterion

10. What is the converse of: "If x equals y , then $a + x$ equals $a + y$ "?

- A) If $a + x$ equals $a + y$, then x equals y

- B) If x does not equal y , then $a + x$ does not equal $a + y$
- C) If $a + x$ does not equal $a + y$, then x equals y
- D) If x equals y , then $a - x$ equals $a - y$

11. What is the status of the proposition and its converse for: "If x equals y , then x^2 equals y^2 "?

- A) Both proposition and converse are true.
- B) Proposition is true, but converse is false (e.g., 2^2 equals 4 , but 2 does not equal 4).
- C) Both are false.
- D) Converse is true, but proposition is false.

12. What is the status of the proposition and its converse for: "If x equals y , then x^3 equals y^3 "?

- A) Both proposition and converse are true.
- B) Proposition is true, converse is false.
- C) Both are false.
- D) Proposition is false, converse is true.

13. What is the converse of: "If n is divisible by 24 , then it is divisible by both 4 and 6 "? Is it true?

- A) Converse is true; statement is false.
- B) Converse is "If n is divisible by 4 and 6 , then it is divisible by 24 "; it is false because 12 is divisible by 4 and 6 but not 24 .
- C) Both statement and converse are true.
- D) Both are false.

14. What is the converse of: "If n is a product of two unequal prime numbers, then it has exactly 4 divisors"?

- A) If n has exactly 4 divisors, then it is a product of two unequal prime numbers.
- B) If n has 3 divisors, it is prime.
- C) If n is a product of primes, it has 2 divisors.

- D) If n has 4 divisors, it is a square number.
15. Find a counterexample to the claim: "All numbers of the form 4 times n squared plus 1 are prime" for small positive integers.
- A) For n equals 1, 5 (prime). For n equals 2, 17 (prime).
 - B) For n equals 4, 4 times 16 plus 1 equals 65, which is 5 times 13 (composite).
 - C) For n equals 1, composite.
 - D) None of these.
16. Consider the claim: "All numbers of the form n squared plus n plus 11 are prime." For what value of n does this fail (giving a composite number)?
- A) n equals 10 (10 squared plus 10 plus 11 equals 121, which is 11 times 11).
 - B) n equals 1 (13).
 - C) n equals 2 (17).
 - D) n equals 0 (11).
17. To check whether a given number is divisible by 8, is it enough to check whether it is divisible by 2 and 4? Why?
- A) Yes, because 2 and 4 are factors of 8.
 - B) No, because numbers like 12 are divisible by 2 and 4, but not by 8.
 - C) Yes, all multiples of 4 are multiples of 8.
 - D) No, 8 requires checking odd numbers.
18. In quadrilateral construction using two sticks as diagonals (type Q), suppose Q satisfies: "If a quadrilateral is of type Q, then it has equal-length diagonals." Should the two sticks be of equal length?
- A) No, they can be different lengths.
 - B) Yes, because the diagonals of the quadrilateral must be equal in length.
 - C) It depends on the angles between sticks.

- D) Sticks have no relation to diagonals.
19. In the same quadrilateral construction with property "If a quadrilateral is of type Q, then it has equal-length diagonals", does it matter how the two sticks are put together?
- A) No, any intersection works.
 - B) Yes, because intersecting them at their midpoints ensures the figure is a parallelogram, rectangle, or square rather than an arbitrary quadrilateral with equal diagonals.
 - C) Intersection point only affects color.
 - D) Sticks must be glued at endpoints.
20. Which of the following describes a situation where both a proposition and its converse are false?
- A) Proposition: "If a number is prime, it is odd." Converse: "If a number is odd, it is prime."
 - B) Proposition: x equals y implies x squared equals y squared.
 - C) Proposition: a squared plus b squared equals c squared implies right triangle.
 - D) Both A and B.

Moderate Level: MCQ Answer Key

1. B | 2. B | 3. A | 4. C | 5. A | 6. B | 7. B | 8. B | 9. C | 10. A | 11. B | 12. A | 13. B | 14. A | 15. B | 16. A | 17. B | 18. B | 19. B | 20. A

Moderate Level: Subjective Questions

1. Explain how Euler disproved Fermat's claim regarding numbers of the form 2 to the power of $(2$ to the power of $n) + 1$.
2. Provide the step-by-step argument given in the text to prove that if a number has an odd number of factors, then it is a perfect square.
3. Why does the argument proving "odd number of factors implies perfect square" not automatically prove the converse (that every square number has an odd number of factors)?
4. How is the proof modified to show that every perfect square has at most and exactly one factor pair with repeating factors?

5. Consider Example 4: "If two triangles have the same area, then they are congruent." Explain why this proposition is false by providing a counterexample.
6. Outline the construction steps used to prove the converse of the Baudhayana-Pythagoras Theorem.
7. In the proof of the converse of the Baudhayana-Pythagoras Theorem, what criteria are used to show that triangle ABC is congruent to triangle XYZ?
8. Frame the converse of: "If n is divisible by 60, then it is divisible by both 5 and 12." Determine if the converse is true and justify.
9. Frame the converse of: "If n is the square of a prime number, then it has exactly 3 factors." Determine if the converse is true and justify.
10. Frame the converse of: "If n is a product of two unequal prime numbers, then it has exactly 4 divisors." Is it true?
11. Find a counterexample to show that the claim "All numbers of the form $4n^2 + 1$ are prime" is false.
12. Find a counterexample to show that the claim "All numbers of the form $n^2 + n + 11$ are prime" is false.
13. Find a counterexample to the claim: "If n is a prime number, then $2^n - 1$ is a prime number."
14. Find a counterexample to the claim: "If n is an even number, then $2^n + 1$ is a prime number."
15. Consider the statement: "If a number is divisible by 8, then it is divisible by both 2 and 4." Justify why this statement is true.
16. Explain why checking divisibility by 2 and 4 is not sufficient to guarantee divisibility by 8, using a specific counterexample.
17. Suppose a category of quadrilaterals Q satisfies: "If a quadrilateral is of type Q , then it has equal-length diagonals." Answer: (a) Should the two sticks be of equal length? Why or why not?
18. For the same category Q in question 17, answer: (b) Will it matter how the two sticks are put together?
19. Suppose category Q satisfies the converse: "If a quadrilateral has equal diagonals, then it is of type Q ." What will be your answers to (a) and (b) now?
20. Give an example from number theory where both a proposition and its converse are false, and explain why both fail.

Moderate Level: Subjective Answers

1. Euler disproved Fermat's claim by showing that for n equals 5, the number 2 to the power of 32 plus 1 is composite rather than prime.
2. Every factor has a partner factor whose product equals the number. If factors pair up normally, the count is even; if a factor pairs with itself, it represents a repeated factor, meaning n equals f times f , which proves n is a perfect square.
3. Because knowing n equals f times f shows a repeating factor pair exists, but a general square number could theoretically have multiple repeating pairs, which would make the total factor count even unless proven otherwise.
4. It is shown that a number cannot have more than one factor pair with repeating factors, thereby ensuring a square number always has an odd number of factors.
5. The proposition is false. Counterexample: Two triangles with completely different shapes and side lengths can easily be constructed to have the exact same area.
6. Given triangle ABC with $a^2 + b^2 = c^2$, we construct another right triangle XYZ with perpendicular sides a and b , and hypotenuse c .
7. We use the Baudhayana-Pythagoras theorem on triangle XYZ to find $c^2 = a^2 + b^2$, so $c = \sqrt{a^2 + b^2}$, and then apply the SSS criterion to show triangle ABC is congruent to triangle XYZ.
8. Converse: "If n is divisible by both 5 and 12, then it is divisible by 60." This is true because 5 and 12 are coprime, so their least common multiple is 60.
9. Converse: "If n has exactly 3 factors, then it is the square of a prime number." This is true because only squares of primes have factors of 1, p , and p^2 , totaling exactly 3 factors.
10. Converse: "If n has exactly 4 divisors, then it is a product of two unequal prime numbers." This is false; counterexample: n equals 8 (2^3), whose divisors are 1, 2, 4, 8, but 8 is a cube of a prime, not a product of two unequal primes.
11. For n equals 4, $4 \times 16 + 1 = 65$, which is composite (5×13).
12. For n equals 10, $10^2 + 10 + 11 = 121$, which is 11×11 (composite).

13. Let n equals 11 (prime). Then 2 to the power of 11 minus 1 equals 2047, which factors as 23 times 89 (composite).
 14. Let n equals 6 (even). Then 2 to the power of 6 plus 1 equals 65, which factors as 5 times 13 (composite).
 15. If a number is divisible by 8, it is a multiple of 8, meaning it is a multiple of both 4 and 2.
 16. It is not sufficient because numbers like 12 or 20 are divisible by 2 and 4, but not divisible by 8.
 17. (a) Yes, the two sticks must be of equal length because the quadrilateral of type Q must have equal-length diagonals.
 18. (b) Yes, it matters how they are put together to form the specific geometric shape.
 19. (a) Yes, the sticks must be of equal length. (b) Yes, how they are assembled determines whether it forms a rectangle, square, or isosceles trapezium.
 20. Proposition: "If a number is prime, it is odd." (False, because 2 is prime and even). Converse: "If a number is odd, it is prime." (False, because 9 is odd and composite).
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DIFFICULT LEVEL

Difficult Level: Multiple Choice Questions (MCQs)

1. What deep mathematical insight does the discussion on factor pairs provide regarding square numbers?
 - A) Square numbers have an infinite number of factors.
 - B) Square numbers possess a unique factor pair (f, f) where the factor repeats, establishing that they have an odd number of factors.
 - C) Square numbers never have prime factors.
 - D) Square numbers can have two distinct factor pairs with repeating factors.

2. In the formal proof that a square number has an odd number of factors, why is it necessary to prove that a number cannot have more than one factor pair with repeating factors?
- A) To prevent the total number of factors from being even.
 - B) Because prime factorizations are unique.
 - C) Because squares cannot be even.
 - D) To calculate the exact square root.
3. Consider Fermat's numbers: 2 to the power of $(2$ to the power of $n)$ plus 1 . For n equals $0, 1, 2, 3$, the resulting values are $3, 5, 17$, and 257 , all of which are prime. Why did Euler's discovery regarding n equals 5 stun the mathematical community?
- A) It proved Fermat was right for all numbers.
 - B) It demonstrated that empirical patterns in number theory can break down at higher values, highlighting the necessity of rigorous proofs over empirical observation.
 - C) It showed that all powers of 2 are composite.
 - D) It proved that 2 to the power of 32 plus 1 is divisible by 3 .
4. When examining the converse of the Baudhayana-Pythagoras theorem, why do we construct a second triangle instead of working solely with the initial triangle ABC?
- A) Because triangle ABC is already known to be a right triangle.
 - B) Because we only know $a^2 + b^2 = c^2$ algebraically and cannot initially assume triangle ABC has a 90 degree angle; constructing a known right triangle provides a benchmark for congruence.
 - C) To prove that all triangles are congruent.
 - D) Because SSS criterion requires two triangles.
5. In the proof of the converse of the Baudhayana-Pythagoras theorem, once we establish $XY^2 = a^2 + b^2$ and use the given condition $a^2 + b^2 = c^2$, what do we deduce about XY ?

- A) XY equals $a + b$
 - B) XY equals c , matching side AB of triangle ABC
 - C) XY equals $2c$
 - D) XY is perpendicular to AB
6. If a proposition states " Y when X ", how should it be written in standard 'If-then' conditional form?
- A) If Y then X
 - B) If X then Y
 - C) If not X then not Y
 - D) X and Y are equivalent
7. Consider Question 8 from Exercise Set 9.1: "If n is divisible by 24, then it is divisible by both 4 and 6." What is the converse and its truth value?
- A) Converse is true; statement is false.
 - B) Converse is "If n is divisible by 4 and 6, then it is divisible by 24"; it is false because 12 is a counterexample.
 - C) Both statement and converse are true.
 - D) Both are false.
8. Consider Question 10 from Exercise Set 9.1: "If n is the square of a prime number, then it has exactly 3 factors." What are the factors of p squared (where p is prime)?
- A) 1, p
 - B) 1, p , p squared
 - C) 1, 2, p , p squared
 - D) p , p squared
9. Consider Question 11 from Exercise Set 9.1: "If n is a product of two unequal prime numbers, then it has exactly 4 divisors." What are the divisors?
- A) 1, p , q

- B) 1, p, q, pq
- C) p, q, pq, p squared q squared
- D) 1, 2, p, q

10. In analyzing Exercise Set 9.1, Question 12: "If n and $n + 3$ have no factors in common, then n is not a multiple of 3." What happens if n is a multiple of 3?

- A) $n + 3$ also becomes a multiple of 3, meaning both share 3 as a common factor, violating the premise. Thus, the proposition is true.
- B) They still have no common factors.
- C) n becomes a prime number.
- D) $n + 3$ is a multiple of 9.

11. Why do mathematicians state "There are no known neat expressions that generate only primes!"?

- A) Because all algebraic expressions always yield composite numbers eventually.
- B) Because expressions like 4 times n squared plus 1 work for many small values of n but eventually produce composite counterexamples.
- C) Because prime numbers cannot be calculated using formulas.
- D) Because prime numbers are infinite.

12. Examine the claim: "If n is a prime number, then 2 to the power of n minus 1 is a prime number." Which prime n provides a counterexample where 2 to the power of n minus 1 is composite?

- A) n equals 2
- B) n equals 3
- C) n equals 11 (2 to the power of 11 minus 1 equals 2047, which is 23 times 89)
- D) n equals 5

13. Examine the claim: "If n is an even number, then 2 to the power of n plus 1 is a prime number." Which even number n provides a counterexample?
- A) n equals 2
 - B) n equals 4
 - C) n equals 6 (2 to the power of 6 plus 1 equals 65 , which is 5 times 13)
 - D) n equals 0
14. In quadrilateral construction, if type Q requires equal-length diagonals, why must the two sticks representing the diagonals be equal in length?
- A) Because diagonals of type Q quadrilaterals are equal by definition of the property.
 - B) To ensure the sticks intersect at 90 degrees.
 - C) Because unequal sticks cannot intersect.
 - D) It is only required for rhombi.
15. In quadrilateral construction, if the property is changed to: "If a quadrilateral has equal diagonals, then it is of type Q ", how does this affect the required length of the sticks?
- A) Sticks must still be of equal length because the hypothesis specifies equal diagonals.
 - B) Sticks can be any length.
 - C) Sticks must be unequal.
 - D) Sticks must form a circle.
16. Which of the following best explains why both a proposition and its converse can be false?
- A) Logical implications do not restrict truth values independently; a statement can falsely link two unrelated conditions, and reversing them can also yield a false link.
 - B) Mathematical laws prevent both from being false.
 - C) Only axioms can be false.

- D) Converses are always the exact negation of propositions.
17. If proposition P is "Line segment AB equals CD" and proposition Q is "Area of triangle ABC equals triangle CDA", what is the logical relationship when testing both directions?
- A) Both are automatically true.
 - B) One may be true while the other is false, demonstrating the asymmetry of converses.
 - C) Both must be false.
 - D) They are inverse statements.
18. In the number theory discussion on factors, why can a square number never have an even number of factors?
- A) Because the square root factor pairs up with itself uniquely, preventing all factors from forming strictly distinct pairs without leaving one self-paired element.
 - B) Because squares are always odd.
 - C) Because factors of squares are always prime.
 - D) This is incorrect; some square numbers do have an even number of factors.
19. What is the fundamental role of a counterexample in advanced mathematical reasoning?
- A) To verify routine calculations.
 - B) To definitively refute universal quantifiers with a single instance, saving proofs from endless validation.
 - C) To generate new theorems.
 - D) To prove converses.
20. When analyzing the statement "If AB equals AC, then IE equals IF" in an incentre configuration, what geometric principle links the side equality to the incentre distance?

- A) SSS congruence and symmetry of angle bisectors in isosceles triangles.
- B) Pythagorean theorem only.
- C) Similarity of right triangles.
- D) Midpoint theorem.

Difficult Level: MCQ Answer Key

1. B | 2. A | 3. B | 4. B | 5. B | 6. B | 7. B | 8. B | 9. B | 10. A | 11. B | 12. C | 13. C | 14. A | 15. A | 16. A | 17. B | 18. A | 19. B | 20. A

Difficult Level: Subjective Questions

1. Construct a rigorous proof demonstrating that if a number n has an odd number of factors, then n must be a perfect square.
2. Explain the subtle flaw that arises if one attempts to prove the converse (that every square number has an odd number of factors) solely by stating that n equals f times f implies a repeating factor pair. How is this flaw resolved?
3. Discuss Euler's refutation of Fermat's prime-generating conjecture. What does this historical example teach us about the limitations of empirical verification in mathematics?
4. Write down the detailed proof of the converse of the Baudhayana-Pythagoras theorem using triangle construction and the SSS congruence criterion.
5. Analyze Exercise 12 from Exercise Set 9.1: Prove or justify why "If n and n plus 3 have no factors in common, then n is not a multiple of 3."
6. Investigate the claim: "All numbers of the form 4 times n squared plus 1 are prime." Show calculations for n equals 1, 2, 3, 4 and identify the counterexample.
7. Investigate the claim: "All numbers of the form n squared plus n plus 11 are prime." Show how it holds for small values and find the exact counterexample.
8. Prove by counterexample that the proposition "If n is a prime number, then 2 to the power of n minus 1 is a prime number" fails for n equals 11. Show the factorization.
9. Prove by counterexample that the proposition "If n is an even number, then 2 to the power of n plus 1 is a prime number" fails for n equals 6. Show the factorization.

10. Consider the statement: "If a number is divisible by 8, then it is divisible by both 2 and 4." Prove algebraically why this holds, and explain why the reverse implication fails.
11. In the context of quadrilateral construction, analyze why the two sticks representing diagonals must be equal in length when the quadrilateral satisfies the property of having equal diagonals.
12. For Exercise 17, analyze whether the intersection point and assembly of the two sticks matter when constructing a specific type of quadrilateral Q with equal diagonals.
13. Give a complete geometric example where both a proposition and its converse are false, providing detailed justification for why neither statement holds.
14. Explain how the phrasing "Y when X" functions logically in mathematical definitions and give an example from the text.
15. Analyze the statement about angle bisectors and incentre I in triangle ABC : Proposition: "If AB equals AC , then IE equals IF ." Explain how symmetry and triangle congruence establish this result.
16. Construct an original number-theoretic proposition where the proposition is true but its converse is false, and provide the exact counterexample for the converse.
17. Explain why finding a counterexample is considered a short and convincing way to disprove a proposition compared to constructing a general disproof.
18. Discuss the logical implications when both a proposition and its converse are true. What term describes such statements?
19. Analyze Question 9 from Exercise Set 9.1: "If n is divisible by 60, then it is divisible by both 5 and 12." State its converse, evaluate its truth, and explain using coprime factor properties.
20. Analyze Question 10 from Exercise Set 9.1: "If n is the square of a prime number, then it has exactly 3 factors." Prove why this is true by examining the divisors of p^2 where p is prime.

Difficult Level: Subjective Answers

1. Every factor d of n has a unique partner n divided by d such that their product equals n . If n has an odd number of factors, pairing them leaves one factor without a distinct partner, meaning it must pair with itself, establishing that n is a perfect square.

2. The flaw is that a square number could theoretically have multiple repeating factor pairs, which would make the total factor count even. This is resolved by proving that a number can have at most one factor pair with repeating factors.
3. Euler's discovery that $2^{32} + 1$ is composite shows that patterns holding for initial terms can fail later, teaching us that empirical verification can never substitute for rigorous mathematical proof.
4. Given triangle ABC with $a^2 + b^2 = c^2$, construct right triangle XYZ with sides a, b and hypotenuse XY. By Pythagoras theorem, $XY^2 = a^2 + b^2 = c^2$, so XY equals c. By SSS criterion, triangle ABC is congruent to triangle XYZ, meaning the angle at C is 90 degrees.
5. If n is a multiple of 3, then n plus 3 is also a multiple of 3, meaning both share 3 as a common factor. By contraposition, if they have no common factors greater than 1, n cannot be a multiple of 3.
6. For n equals 1, 5; for n equals 2, 17; for n equals 3, 37; for n equals 4, 65, which is composite (5 times 13).
7. For n equals 1, 2, 3, values are prime. For n equals 10, $10^2 + 10 + 11 = 121$, which is 11 times 11 (composite).
8. For n equals 11, $2^{11} - 1 = 2047$, which factors as 23 times 89.
9. For n equals 6, $2^6 + 1 = 65$, which factors as 5 times 13.
10. Algebraic proof: If n is divisible by 8, $n = 8k = 4 \times 2k = 2 \times 4k$, showing divisibility by 4 and 2. The reverse fails because 12 is divisible by 2 and 4, but not by 8.
11. By definition of the quadrilateral property having equal diagonals, the two constituent diagonal sticks must be equal in length to form the geometric constraints.
12. Yes, the intersection point matters greatly; assembling them at their exact midpoints versus off-center determines whether the resulting figure is a parallelogram, rectangle, or square versus an asymmetric trapezium.
13. Proposition: "If a quadrilateral is a square, then it is a kite." (False). Converse: "If a quadrilateral is a kite, then it is a square." (False). Both fail.
14. "Y when X" expresses the exact same logical implication as "If X then Y." Example: "A number has an odd number of factors when it is a perfect

square" means "If a number is a perfect square, then it has an odd number of factors."

15. If AB equals AC , the triangle is isosceles. Angle bisectors meeting at incentre I create symmetrical congruence triangles containing segments IE and IF , proving IE equals IF .
 16. Proposition: "If a number is a multiple of 10, then it is a multiple of 5." (True). Converse: "If a number is a multiple of 5, then it is a multiple of 10." (False; counterexample: 15).
 17. A counterexample exhibits a concrete, undeniable violation of a universal statement with a single calculation, whereas a general proof requires demonstrating validity across an infinite domain.
 18. When both a proposition and its converse are true, the statements are said to be logically equivalent or to imply each other.
 19. Converse: "If n is divisible by both 5 and 12, then it is divisible by 60." True, because 5 and 12 are coprime, meaning their LCM is 60.
 20. Let n equal p squared where p is prime. The positive divisors of p squared are strictly 1, p , and p squared, yielding exactly 3 divisors.
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OLYMPIAD LEVEL

Olympiad Level: Multiple Choice Questions (MCQs)

1. Consider generalized numbers of the form N equals p sub 1 to the power of a sub 1 times p sub 2 to the power of a sub 2 up to p sub k to the power of a sub k . What is the exact condition on the exponents for N to have an odd number of divisors?
 - A) All exponents must be odd integers.
 - B) All exponents must be even integers, which precisely characterizes all perfect squares.
 - C) The sum of exponents must be prime.
 - D) At least one exponent must be zero.
2. Fermat numbers are defined as F sub n equals 2 to the power of (2 to the power of n) plus 1. If we evaluate the proposition: "For all integers n

greater than or equal to 0, $F \leq n$ is prime," and its converse, how do their truth values stand after Euler's discovery?

- A) Both proposition and converse are true.
 - B) Proposition is false, while the converse is also false.
 - C) Proposition is true; converse is false.
 - D) Both are true for n less than or equal to 4.
3. In number theory, let τ of n denote the number of divisors of n . The proposition states: "If τ of n is odd, then n is a square." What can be said about the converse: "If n is a square, then τ of n is odd"?
- A) False, squares can have even divisors.
 - B) True, proven by establishing that squares have a unique self-paired factor and no more than one such repeating pair.
 - C) True only for prime squares.
 - D) False, because factor pairs can repeat infinitely.
4. Consider the Baudhayana-Pythagoras theorem and its converse. In non-Euclidean geometries, does the converse hold?
- A) Yes, it holds universally across all geometric systems.
 - B) No, because the Pythagorean theorem itself takes a different algebraic form in non-Euclidean spaces, making the Euclidean converse specific to Euclidean flat space.
 - C) Only in hyperbolic geometry.
 - D) It holds if triangles are equilateral.
5. Let P be the proposition: "If n is divisible by a times b times c (where a, b, c are pairwise coprime), then n is divisible by a, b , and c ." What is the strongest statement regarding its converse?
- A) The converse is always true without any conditions.
 - B) The converse is true if and only if a, b, c are pairwise coprime; otherwise, it fails.
 - C) The converse is always false.
 - D) The converse holds only for prime numbers.

6. In the context of prime-generating polynomials like P of n equals $n^2 + n + 11$, for what range of consecutive integers n does P of n yield only prime numbers before encountering its first composite counterexample?
- A) n equals 0 to 9
 - B) n equals 0 to 40
 - C) n equals 1 to 100
 - D) Infinite primes
7. If a quadrilateral is constructed using two diagonals of lengths d_1 and d_2 intersecting at angle θ , its area is given by $\frac{1}{2} d_1 d_2 \sin \theta$. If category Q requires equal diagonals and maximum area for a fixed diagonal length, what must θ be?
- A) θ equals 0 degrees
 - B) θ equals 90 degrees; orthogonal intersection maximizes area and defines classes like squares and rectangles.
 - C) θ equals 60 degrees
 - D) Angle does not affect area.
8. Consider the logical biconditional statement. Which of the following pairs forms a valid biconditional statement from the text?
- A) " n is a perfect square" and " n has an odd number of factors"
 - B) "Two triangles have the same area" and "Two triangles are congruent"
 - C) "A number is a multiple of 3" and "A number is a multiple of 6"
 - D) "Quadrilateral is a square" and "Quadrilateral has equal angles"
9. Analyze the number-theoretic claim in Exercise 11: "If n is a product of two unequal prime numbers, then it has exactly 4 divisors." If we take the converse, why does n equals 8 serve as a counterexample?
- A) Because the divisors of 8 are 1, 2, 4, 8 (count equals 4), but 8 is 2 cubed (cube of a prime, not product of two unequal primes).

- B) Because 8 has 5 divisors.
 - C) Because 8 is prime.
 - D) Because 8 is a product of unequal primes.
10. In rigorous proof writing, suppose we wish to prove both a proposition and its converse. What proof architecture is traditionally employed in advanced mathematics?
- A) Proof by contradiction only
 - B) Necessity and Sufficiency (or If and only if / bidirectional proof)
 - C) Mathematical induction on number of sides
 - D) Constructive counterexample generation
11. What is the precise number of divisors for n equals p to the power of a times q to the power of b where p and q are distinct primes?
- A) a plus b
 - B) $(a + 1)$ times $(b + 1)$
 - C) a times b
 - D) 2 to the power of $a + b$
12. Using the divisor formula, what condition on a and b guarantees that n equals p to the power of a times q to the power of b has an odd number of divisors?
- A) Both a and b must be even integers, which forces n to be a perfect square.
 - B) Both a and b must be odd.
 - C) a is even, b is odd.
 - D) Divisor count can never be odd for composite numbers.
13. In the context of Mersenne primes, the text explores the proposition: "If n is prime, then 2 to the power of n minus 1 is prime." While true for small primes, it fails for n equals 11 . What does this specific failure illustrate?
- A) Heuristics are infallible.

- B) Small-case verification is insufficient because primality tests can break down due to hidden algebraic factorizations at larger prime indices.
- C) $2^n - 1$ is never prime for composite numbers.
- D) Mersenne primes only exist for even exponents.

14. Examine the incentre configuration where $AB = AC$ implies $IE = IF$. If we consider the converse, how does the proof proceed in advanced geometry?

- A) By reversing the angle-chasing and congruence arguments, establishing that the converse is also true.
- B) The converse is false because incentres are independent of side lengths.
- C) By applying the Pythagorean theorem directly to the sides.
- D) By assuming the triangle is equilateral.

15. What structural insight is gained from studying propositions and their converses at the Grade 9 level in Ganita Manjari?

- A) It introduces students to the foundational logic of mathematical implication, bidirectional reasoning, equivalence, and the supreme importance of counterexamples.
- B) It focuses solely on arithmetic calculations.
- C) It replaces algebra with geometry.
- D) It teaches calculus foundations.

16. If a proposition is true, what can be rigorously deduced about its contrapositive?

- A) It is always false.
- B) It is logically equivalent and always true.
- C) It is the converse.
- D) It has no relation to the original proposition.

17. In the analysis of polynomial generators like $4n^2 + 1$ or $n^2 + n + 11$, why do algebraic polynomials with integer coefficients fail to generate all prime numbers?

- A) Due to modular arithmetic periodicities, polynomial sequences eventually hit composite values governed by fixed factor thresholds.
- B) Because polynomials cannot produce numbers greater than 100.
- C) Because prime numbers are random.
- D) Because negative integers invalidate them.

18. In quadrilateral stick construction, if the category Q is defined such that both the proposition and its converse hold true, what geometric class of quadrilaterals does Q represent?

- A) Scalene trapezia
- B) Rectangles (and squares as a subset)
- C) Kites
- D) Parallelograms

19. When evaluating mathematical arguments like the factor-partner pair proof, what logical fallacy would be committed if one assumed that proving the converse automatically proves the proposition?

- A) Fallacy of affirming the consequent.
- B) Circular reasoning.
- C) Division by zero.
- D) False equivalence.

20. What is the ultimate mathematical takeaway regarding the interaction between theorems and their converses as demonstrated across Chapter 9?

- A) Theorems and converses are always identical in scope.
- B) A theorem's truth bears no logical necessity on the truth of its converse; each direction requires independent rigorous verification or a distinct counterexample.
- C) Converses are always easier to prove than original propositions.

- D) Geometry has converses, but number theory does not.

Olympiad Level: MCQ Answer Key

1. B | 2. B | 3. B | 4. B | 5. B | 6. A | 7. B | 8. A | 9. A | 10. B | 11. B | 12. A | 13. B | 14. A | 15. A | 16. B | 17. A | 18. B | 19. A | 20. B

Olympiad Level: Subjective Questions

1. Prove rigorously using fundamental prime factorization theorem that a positive integer n has an odd number of divisors if and only if n is a perfect square.
2. Analyze the limitations of empirical pattern recognition in number theory, citing Euler's refutation of Fermat's prime numbers and discuss how this elevates the necessity of formal converse testing.
3. Provide a complete, rigorous vector or coordinate geometry proof of the converse of the Baudhayana-Pythagoras theorem.
4. Investigate the generalized proposition: "If n is divisible by a times b times c , then n is divisible by a , b , and c ." Formulate its converse, prove why the converse requires pairwise coprimality, and construct a counterexample when coprimality fails.
5. Explore the polynomial prime generator P of n equals $n^2 + n + 11$. Prove mathematically why it yields primes for n equals 0 through 9, and demonstrate step-by-step why it fails at n equals 10.
6. Conduct an advanced divisibility analysis for Exercise Set 9.1, Question 12: Prove that if n and $n + 3$ share no common factors greater than 1, then n is not a multiple of 3.
7. In the context of quadrilateral diagonal construction, formulate a rigorous proof explaining how the orthogonal intersection combined with equal diagonal lengths characterizes rectangles and squares.
8. Prove the biconditional equivalence for the statement involving AB equals AC and IE equals IF in triangle ABC , providing both forward and converse proofs.
9. Analyze Mersenne numbers. Formulate the proposition "If n is prime, then $2^n - 1$ is prime", explain why testing small primes creates a false sense of universal truth, and analyze the counterexample at n equals 11.

10. Examine the number of divisors formula. Use this formula to prove that the divisor count is odd if and only if all exponents are even, connecting back to perfect squares.
11. Investigate the false converse of Question 11: "If n has exactly 4 divisors, then n is the product of two unequal prime numbers." Classify all possible integer forms for n such that $\tau(n) = 4$.
12. Define the logical fallacy of affirming the consequent in the context of mathematical propositions and their converses. Provide an original mathematical example.
13. Discuss the epistemological role of counterexamples in mathematics as highlighted in Chapter 9. Contrast how a single counterexample instantly invalidates a universal proposition versus the exhaustive nature required for a direct proof.
14. Analyze the statement: "There are no known neat expressions that generate only primes!" Review historical attempts and explain using elementary number theory why polynomial expressions cannot exclusively generate primes.
15. Prove that if x and y are real numbers, the proposition $x = y$ implies $x^3 = y^3$ is true, and its converse is also true. Contrast this with the even power case.
16. Construct an original Olympiad-level geometric proposition regarding cyclic quadrilaterals, state its converse, and determine the truth value of both.
17. Analyze the statement "Y when X". Translate three complex mathematical definitions from algebra or geometry into this phrasing and demonstrate how identifying hypothesis and conclusion prevents converse confusion.
18. Examine the structural proof of Example 3. Address the subtle edge case where a number might possess multiple factor pairs of the form (f, f) , and explain how the text resolves this.
19. Formulate a comprehensive comparative analysis of the four logical combinations of a proposition and its converse, providing a mathematical example for each quadrant.
20. Synthesize the core pedagogical objective of Chapter 9: How does training Grade 9 students in conditional logic, converse formulation, and counterexample construction lay the groundwork for rigorous theorem proving in higher mathematics?

Olympiad Level: Subjective Answers

1. Let n be factored into primes. The total number of divisors is the product of (exponents plus 1). For this product to be odd, every exponent plus 1 must be odd, meaning every exponent is even. This establishes that n is a perfect square.
2. Empirical pattern recognition relies on observing finite instances. Euler's counterexample 2 to the power of 32 plus 1 being composite shattered empirical induction, proving that heuristic patterns cannot replace deductive proofs.
3. Using vector dot products, let vectors a and b form sides meeting at angle θ , with c equal to a minus b . Expanding magnitude squared and equating to a squared plus b squared forces $\cos \theta$ to equal 0 , yielding θ equal to 90 degrees.
4. The converse requires pairwise coprimality. Counterexample: Let a equals 2 , b equals 4 , c equals 2 . A number n equals 4 is divisible by 2 and 4 , but not by 16 .
5. For n equals 0 to 9 , values are prime. At n equals 10 , 10 squared plus 10 plus 11 equals 121 , which is 11 times 11 (composite).
6. Assume n is a multiple of 3 . Then n plus 3 is also a multiple of 3 , sharing 3 as a common factor. By contraposition, if they have no common factors, n cannot be a multiple of 3 .
7. Orthogonal intersection of two equal diagonals maximizes area and enforces symmetry constraints yielding rectangles and squares. Off-center intersection breaks symmetry, resulting in general trapezia.
8. Forward proof uses isosceles triangle symmetry and angle bisectors. Converse proof uses equal distances from incentre to sides to establish base angle equality and side equality.
9. Testing n equals $2, 3, 5, 7$ yields primes, creating false inductive certainty. At n equals 11 , 2 to the power of 11 minus 1 equals 2047 , which factors as 23 times 89 .
10. The divisor product formula is odd if and only if each factor is odd, which forces every prime exponent to be even, defining a perfect square.
11. Tau of n equals 4 can arise from $(3 \text{ plus } 1) \text{ equal to } 4$ (giving n equals p cubed) or $(1 \text{ plus } 1) \text{ times } (1 \text{ plus } 1) \text{ equal to } 4$ (giving n equals p times q). Thus n can be pq or p cubed.

12. Affirming the consequent is inferring that if X implies Y is true, then Y implies X must be true. Example: Confusing "squares have 4 equal sides" with "shapes with 4 equal sides are squares."
13. A counterexample provides a single definitive refutation destroying a universal quantifier instantly, whereas a direct proof requires infinite domain validation.
14. Polynomials with integer coefficients cannot generate exclusively primes because modular periodicity ensures the sequence eventually generates composite multiples.
15. Cubing is one-to-one for reals. For even powers, x squared equals y squared permits x equals plus or minus y , allowing two solutions which explains why even power converses fail without absolute value constraints.
16. Proposition regarding cyclic quadrilateral angle properties and its converse both hold true under strict Euclidean circle constraints.
17. "Y when X" means "If X then Y." Examples include divisibility rules, equilateral triangle definitions, and matrix invertibility conditions.
18. The text resolves the edge case of multiple repeating factor pairs by proving algebraically that a number can possess at most one such factor pair with repeating factors.
19. (1) Both true: Pythagoras and converse. (2) Both false: Prime and odd number statements. (3) Prop true / Conv false: Multiples of 6 and 3. (4) Prop false / Conv true: Rectangle and square implications.
20. Chapter 9 builds mathematical maturity by teaching directional logic, converse formulation, logical equivalence, and counterexample skepticism.