

NCERT Class 9th Mathematics (Ganit Manjiri)

Chapter:7 The Mathematics of Maybe: Introduction to Probability

This comprehensive question bank is designed in accordance with the NCERT Class 9 Mathematics curriculum. It covers the fundamental concepts of experimental (or empirical) probability, including outcomes, trials, sample events, and the calculation of probabilities based on frequency data.

Section A: Multiple Choice Questions (20 Easy MCQs) Alpha Level

1. The probability of an event can never be:
A) 0
B) 1
C) 0.5
D) -0.5
2. If the probability of an event happening is 0.35, what is the probability of it not happening?
A) 0.65
B) 0.35
C) 0.55
D) 1
3. A coin is tossed 100 times and head appear 64 times. The probability of getting a tail is:
A) $\frac{64}{100}$
B) $\frac{36}{100}$
C) $\frac{1}{2}$
D) 1

4. The sum of the probabilities of all possible outcomes of an experiment is:
- A) 0
 - B) 0.5
 - C) 1
 - D) Infinity
5. An event that is certain to happen has a probability of:
- A) 0
 - B) 0.5
 - C) 1
 - D) 100
6. An impossible event has a probability of:
- A) 1
 - B) 0
 - C) -1
 - D) 0.1
7. A die is thrown once. The probability of getting an even number is:
- A) $\frac{1}{2}$
 - B) $\frac{1}{3}$
 - C) $\frac{1}{6}$
 - D) $\frac{2}{3}$
8. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.
- A) $\frac{1}{5}$
 - B) $\frac{4}{5}$
 - C) $\frac{3}{5}$
 - D) $\frac{2}{5}$
9. Which of the following values cannot be the probability of an event?
- A) $\frac{2}{3}$

- B) 15%
- C) 0.7
- D) $\frac{3}{2}$

10. A bag contains 5 red and 3 blue balls. If a ball is drawn at random, the probability of getting a red ball is:

- A) $\frac{3}{8}$
- B) $\frac{5}{8}$
- C) $\frac{1}{5}$
- D) $\frac{1}{3}$

11. Two coins are tossed simultaneously. The total number of possible outcomes is:

- A) 2
- B) 4
- C) 6
- D) 8

12. Three coins are tossed simultaneously. The total number of possible outcomes is:

- A) 3
- B) 6
- C) 8
- D) 9

13. When a single die is thrown, the total number of possible outcomes is:

- A) 1
- B) 2
- C) 4
- D) 6

14. In 50 tosses of a coin, the tail is obtained 22 times. The empirical probability of getting a head is:

- A) $\frac{22}{50}$
- B) $\frac{28}{50}$
- C) $\frac{1}{2}$

D) $25/50$

15. The probability of drawing a king from a well-shuffled deck of 52 playing cards is:

A) $1/13$

B) $4/13$

C) $1/52$

D) $1/26$

16. If $P(E) = 0.05$, then the probability of 'not E' is:

A) -0.05

B) 0.5

C) 0.95

D) 1.05

17. A letter is chosen at random from the word 'MATHEMATICS'. The probability that it is a vowel is:

A) $4/11$

B) $7/11$

C) $3/11$

D) $5/11$

18. The probability of getting a number greater than 4 when a die is rolled is:

A) $1/3$

B) $2/3$

C) $1/2$

D) $1/6$

19. A survey of 200 students showed that 140 like statistics. The probability that a student chosen at random likes statistics is:

A) $7/10$

B) $3/10$

C) $1/2$

D) $14/200$

20. The probability of an event E satisfies which of the following inequalities?

A) $0 < P(E) < 1$

B) $0 \leq P(E) \leq 1$

C) $P(E) > 0$

D) $P(E) \leq 1$

MCQ Answer Key

1-D, 2-A, 3-B, 4-C, 5-C, 6-B, 7-A, 8-B, 9-D, 10-B, 11-B, 12-C, 13-D, 14-B, 15-A, 16-C, 17-A, 18-A, 19-A, 20-B.

Section B: Subjective Questions (20 Easy Subjective Questions) Alpha Level

1. Define experimental probability in your own words.
2. A coin is tossed 200 times and heads are obtained 115 times. Find the experimental probability of getting a tail.
3. Write down all the possible outcomes when two distinct coins are tossed together.
4. A die is thrown 60 times and a 5 is obtained 12 times. What is the empirical probability of getting a 5?
5. If the probability of winning a game is 0.73, what is the probability of losing it?
6. A bag contains 4 white, 5 black, and 6 green balls. If one ball is drawn at random, find the probability that it is black.
7. A student throws a die 100 times and gets the face 6 twenty-five times. Find the probability of not getting a 6.
8. In a class of 40 students, 24 are girls. Find the probability that a student selected at random is a boy.
9. Explain why the probability of an event cannot be 1.2.
10. A box contains cards numbered from 1 to 15. One card is drawn at random. Find the probability that the number on the card is an even number.
11. During a medical check-up of 50 students, 15 were found to have blood group A. Find the probability that a randomly selected student has blood group A.
12. A weather forecast department accurately predicted the weather on 180 days out of 300 days. What is the probability that the prediction was incorrect on a given day?
13. A wheel has 8 equal sectors colored Red, Blue, Green, Yellow, Orange, Purple, White, and Black. If spun once, what is the probability of pointing to the Red sector?
14. State the value of the probability of a sure event and an impossible event.

15. A factory manufactured 1000 bulbs and found 45 to be defective. Find the probability of choosing a non-defective bulb.
16. In a survey of 150 people, 90 prefer drinking tea over coffee. Find the probability that a selected person prefers coffee.
17. What is the sum of the probability of an event happening and the probability of it not happening?
18. A letter is chosen at random from the English alphabet. Find the probability that it is a consonant.
19. A coin is tossed 10 times and tail appears 4 times. What is the empirical probability of getting a head?
20. If a die is thrown, what is the probability of getting a number less than 7? Explain your answer.

Section A: Multiple Choice Questions (20 Moderate MCQs) Beta Level

21. A coin is tossed 500 times. If the probability of getting a head is 0.42, how many times does a tail appear?
 - A) 210
 - B) 290
 - C) 250
 - D) 180
22. Out of the first 50 natural numbers, a number is chosen at random. The probability that it is a prime number is:
 - A) $\frac{3}{10}$

- B) $\frac{7}{25}$
- C) $\frac{4}{25}$
- D) $\frac{8}{25}$

23. Two coins are tossed simultaneously 300 times. Two tails appear 75 times. The probability of getting at least one head is:

- A) $\frac{1}{4}$
- B) $\frac{3}{4}$
- C) $\frac{1}{2}$
- D) $\frac{2}{3}$

24. A die is rolled 120 times and a composite number is obtained 48 times. The empirical probability of getting a prime number or 1 is:

- A) $\frac{2}{5}$
- B) $\frac{3}{5}$
- C) $\frac{1}{2}$
- D) $\frac{1}{5}$

25. A bag contains 6 red, 4 blue, and some black balls. If the probability of drawing a blue ball is $\frac{1}{4}$, find the total number of black balls.

- A) 4
- B) 6
- C) 8
- D) 10

26. In a commercial batch of 2000 light bulbs, 100 are defective. If a sample of 5 bulbs is chosen, which of the following is true about the sum of probabilities of all possible defective bulbs (0, 1, 2, 3, 4, 5)?

- A) It is less than 1
- B) It is greater than 1
- C) It is exactly 1
- D) It depends on the sample

27. Three coins are tossed simultaneously 200 times. No head appears 32 times. The

probability of getting at least one head is:

- A) $42/50$
- B) $8/25$
- C) $17/25$
- D) $21/25$

28. A number is selected at random from the integers 1 to 30. The probability that it is a multiple of both 2 and 3 is:

- A) $1/5$
- B) $1/6$
- C) $1/10$
- D) $1/3$

29. In a survey of 360 households, 240 households have 2 children, 90 have 1 child, and 30 have no children. The probability that a randomly chosen household has at most 1 child is:

- A) $1/3$
- B) $2/3$
- C) $1/4$
- D) $3/4$

30. A letter is chosen at random from the word "PROBABILITY". The probability that it is a vowel is:

- A) $3/11$
- B) $4/11$
- C) $5/11$
- D) $2/11$

31. A card is drawn from a well-shuffled pack of 52 cards. The probability that it is either a red card or a king is:

- A) $7/13$
- B) $1/2$
- C) $15/26$
- D) $8/13$

32. The probability of an event happening is expressed as p . The probability of its complementary event will satisfy:
- A) Always equal to p
 - B) $1 - p$
 - C) $p - 1$
 - D) $\frac{1}{p}$
33. A die is thrown once. The probability of getting a perfect square number is:
- A) $\frac{1}{3}$
 - B) $\frac{1}{2}$
 - C) $\frac{1}{6}$
 - D) $\frac{2}{3}$
34. From a deck of 52 cards, all the face cards are removed. If a card is drawn from the remaining deck, the probability that it is an ace is:
- A) $\frac{1}{13}$
 - B) $\frac{1}{10}$
 - C) $\frac{4}{40}$
 - D) $\frac{2}{5}$
35. A spinner is divided into 12 equal sectors numbered 1 to 12. The probability that the spinner lands on a number which is a factor of 12 is:
- A) $\frac{1}{2}$
 - B) $\frac{5}{12}$
 - C) $\frac{7}{12}$
 - D) $\frac{2}{3}$
36. Two dice are thrown together. The probability that the sum of the two numbers appearing on the top faces is 10 is:
- A) $\frac{1}{12}$
 - B) $\frac{1}{18}$
 - C) $\frac{1}{9}$
 - D) $\frac{1}{6}$

37. A month is chosen at random from a standard leap year. The probability that the month has exactly 30 days is:

- A) $\frac{1}{3}$
- B) $\frac{5}{12}$
- C) $\frac{1}{4}$
- D) $\frac{7}{12}$

38. In 1000 trials of an experiment, an event E happens 345 times. The probability of event not happening is:

- A) 0.345
- B) 0.655
- C) 0.555
- D) 0.645

39. The table shows the marks obtained by 80 students in a test:

Marks	0-20	20-40	40-60	60-80
Students	12	28	32	8

40. The probability that a student chosen at random scored 40 or more marks is:

- A) $\frac{1}{2}$
- B) $\frac{2}{5}$
- C) $\frac{3}{5}$
- D) $\frac{7}{10}$

41. A box contains 90 discs numbered 1 to 90. If one disc is drawn at random, the probability that it bears a two-digit number is:

- A) $\frac{81}{90}$
- B) $\frac{9}{90}$
- C) $\frac{80}{90}$
- D) $\frac{89}{90}$

MCQ Answer Key

One-Line Format Key: 1-B, 2-A, 3-B, 4-B, 5-B, 6-C, 7-D, 8-A, 9-A, 10-B, 11-A, 12-B, 13-A, 14-B, 15-A, 16-A, 17-A, 18-B, 19-A, 20-A.

Section B: Subjective Questions (20 Moderate Subjective Questions) Beta Level

21. A coin is tossed 400 times and the outcomes are noted. The empirical probability of getting a head is 0.45. Find the actual frequency of tails obtained during this experiment.
22. Over the past 200 working days, the weather forecasts of a local station were correct 160 times. On picking a day at random from the past record, what is the probability that the forecast was not correct?
23. A bag contains 5 red balls, 8 white balls, and 4 green balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is neither red nor green.
24. Two coins are tossed simultaneously 500 times. We get two heads 105 times, one head 275 times, and no head 120 times. Find the probability of occurrence of each of these events and show that their sum is 1.
25. Three coins are tossed simultaneously 150 times with the following frequencies of different outcomes:
 - 3 heads: 25 times
 - 2 heads: 55 times
 - 1 head: 45 times
 - No head: 25 timesIf the three coins are simultaneously tossed again, compute the probability of getting more than 1 head.
26. A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table:

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

27. Find the probability of getting a prime number outcome.
28. The percentages of marks obtained by a student in monthly unit tests are given below:

Unit Test	I	II	III	IV	V
% of marks	69	71	73	68	74

29. Based on this data, find the probability that the student gets more than 70% marks in a unit test chosen at random.
30. An insurance company selected 2000 drivers at random in a particular city to find a relationship between age and car accidents. The data obtained is given in the following table:

Age of Drivers (in years)	Accidents in one year			
	0	1	2	more than 2
18-29	440	160	110	35
30-50	505	125	60	22
Above 50	360	45	35	8

31. Find the probability of a driver chosen at random from the city being 18–29 years of age and having exactly 2 accidents in one year.
32. Referring to the table in question 8, find the probability of a driver chosen at random being 30–50 years of age and having no accidents in a year.
33. A multi-national firm surveyed 5000 families to study their choice of cars. The data is as follows:
- Families owning Type A car only: 1200
 - Families owning Type B car only: 1800

- Families owning both Type A and Type B cars: 800
- Families owning neither car: 1200

Find the probability that a family chosen at random owns at least one type of car.

34. From a deck of 52 playing cards, all spades are removed. If a card is drawn at random from the remaining deck, what is the probability that it is a Queen?
35. A box contains 50 tokens numbered 1 to 50. A token is picked at random. Find the probability that the token number is divisible by 4 or 5.
36. A die is rolled. Find the probability of getting a number which is a multiple of 2 but not a multiple of 3.
37. In a cricket match, a batsman plays 40 balls and hits a boundary 8 times. Find the probability that he does not hit a boundary in the next ball, assuming his scoring pattern follows the same experimental behavior.
38. Cards marked with numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is a perfect cube.
39. Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardized conditions favorable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Bag	1	2	3	4	5
No. of germinated seeds	40	48	42	39	41

40. What is the probability of selecting a bag which has more than 40 germinated seeds?
41. A dynamic survey of 400 public workers showed that 260 workers favor a new policy while the rest oppose it. Find the probability that a worker chosen at random opposes the policy.
42. When 3 coins are tossed simultaneously, write down all the possible items in the sample space. Find the probability of getting exactly two tails experimentally if it aligns with theoretical probability.
43. A standard bag contains 15 marbles, some of which are green and the remaining are blue. If the probability of drawing a blue marble is $\frac{2}{3}$, determine the total number of green marbles in the bag.

44. The blood groups of 30 students of Class IX are recorded as follows:

A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O, A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.

Represent this data in the form of a frequency distribution table and calculate the probability that a student selected at random has blood group AB.

Section A: Multiple Choice Questions (20 Difficult MCQs) Delta Level

42. A non-standard six-sided die has its faces marked with numbers 1, 2, 2, 3, 3, 3. If this die is rolled 600 times, and the experimental outcomes perfectly match the theoretical expectations, what is the empirical probability of rolling a number that is a prime factor of 12?

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{5}{6}$

D) $\frac{2}{3}$

43. Three coins are tossed simultaneously 800 times. The outcomes are recorded as follows: 3 Heads (110 times), 2 Heads (290 times), 1 Head (310 times), and No Head (90 times). What is the empirical probability that the number of heads obtained differs from 1 by an absolute value of at least 1?

A) $\frac{31}{80}$

B) $\frac{49}{80}$

C) $\frac{3}{8}$

D) $\frac{5}{8}$

44. A bag contains x red, $2x$ blue, and $3x$ green balls. A ball is drawn, its color noted, and replaced. This is done 120 times, yielding 24 red balls. If 60 more trials are performed, what is the expected cumulative frequency of getting a blue ball, assuming total empirical convergence to the initial contents ratio?

A) 40

B) 60

C) 36

D) 48

45. A four-digit number is selected at random from all four-digit positive integers. What is the probability that the sum of its digits is exactly 3?
- A) $1/1500$
 - B) $1/900$
 - C) $1/1000$
 - D) $7/9000$
46. In an experiment involving 1000 trials of rolling a pair of dice, the sum of the top faces is monitored. The event A is defined as "the sum is a multiple of 3 or 4". If the experiment yields results matching standard theoretical distributions, the empirical probability $P(A)$ is closest to:
- A) $7/12$
 - B) $5/12$
 - C) $4/9$
 - D) $5/9$
47. A point is chosen at random inside a square of side length 4 units containing a circle touching all four sides. If 500 random points are plotted, and 393 fall inside the circle, what is the empirical value of π derived from this experiment?
- A) 3.144
 - B) 3.141
 - C) 3.122
 - D) 3.168
48. From a thoroughly shuffled pack of 52 cards, two cards are drawn sequentially without replacement. If the first card drawn was a King, what is the experimental probability that the second card drawn is also a face card (King, Queen, or Jack)?
- A) $11/51$
 - B) $12/51$
 - C) $11/52$
 - D) $12/52$
49. An integer n is chosen at random from the range $1 \leq n \leq 100$. What is the probability that $n + \frac{120}{n}$ is an integer?

- A) $\frac{3}{25}$
- B) $\frac{4}{25}$
- C) $\frac{14}{100}$
- D) $\frac{16}{100}$

50. The probability of a runner winning an experimental race track simulation under muddy conditions is denoted by $\frac{a}{a+b}$ and losing is $\frac{b}{a+b}$. If the odds change such that the probability of winning doubles, the new probability of losing is expressed as:

- A) $\frac{(b-a)}{(a+b)}$
- B) $\frac{b}{2a+b}$
- C) $\frac{b-a}{2a+b}$
- D) $\frac{b}{2(a+b)}$

51. A digital counter randomly generates 3-digit binary strings (from 000 to 111) with equal frequency over 8000 runs. What is the empirical probability that a generated string contains more 1s than 0s?

- A) $\frac{1}{4}$
- B) $\frac{3}{8}$
- C) $\frac{1}{2}$
- D) $\frac{5}{8}$

52. A group of 1200 people were classified by age and preference for three mobile apps (X, Y, Z):

Age / Preference	App X	App Y	App Z
Under 25	150	280	170

Age / Preference	App X	App Y	App Z
25 and Above	210	190	200

53. What is the probability that a randomly picked individual is either under 25 years old or prefers App Y?
- A) $79/120$
 B) $60/120$
 C) $47/120$
 D) $98/120$
54. Two large boxes contain colored balls. Box 1 has 3 Red and 2 Blue. Box 2 has 2 Red and 4 Blue. A fair coin is tossed; if Heads, a ball is drawn from Box 1, if Tails, from Box 2. Over 600 complete iterations, how many times do you empirically expect to draw a Red ball?
- A) 260
 B) 280
 C) 300
 D) 320
55. An empirical study tracks the daily attendance of 50 teachers over 200 school days. The event E signifies "fewer than 45 teachers are present". If E occurred on 14 days, the probability that on a randomly selected day at least 45 teachers are working is:
- A) 0.07
 B) 0.93
 C) 0.86
 D) 0.14
56. A complete alphabet set of plastic letters {A, B, C, ..., Z} is placed in a pouch. A letter is drawn 260 times with replacement, and a consonant is drawn 190 times. What is the absolute difference between this experimental probability and the standard theoretical probability of drawing a consonant?
- A) $1/26$

- B) $\frac{3}{26}$
- C) $\frac{5}{26}$
- D) 0

57. A specialized wheel is indexed with integers from -5 to +5 (including 0). If spun 1100 times, the arrow lands on negative integers 480 times and on zero 100 times. What is the empirical probability of landing on a positive number?

- A) $\frac{26}{55}$
- B) $\frac{24}{55}$
- C) $\frac{12}{55}$
- D) $\frac{29}{55}$

58. A collection of 60 cards is numbered 1 to 60. A card is drawn at random. What is the probability that the number on the card is divisible by 3 or 5 but not divisible by 15?

- A) $\frac{7}{15}$
- B) $\frac{2}{5}$
- C) $\frac{1}{3}$
- D) $\frac{8}{15}$

59. The letters of the word "PERMUTATION" are written on separate slips and mixed in a bowl. One slip is drawn at random. The probability that the slip bears a letter that appears more than once in the word is:

- A) $\frac{2}{11}$
- B) $\frac{4}{11}$
- C) $\frac{3}{11}$
- D) $\frac{1}{11}$

60. A loaded six-sided die is constructed such that the probability of an even face turning up is twice the probability of an odd face turning up. If this die is rolled 900 times, what is the expected frequency of getting a prime number?

- A) 300
- B) 400
- C) 500
- D) 450

61. In an encryption game, a 2-digit number (10 to 99) is randomly transmitted. What is the probability that the transmitted number is a prime number whose unit digit is also a prime number?
- A) 11/90
B) 13/90
C) 2/15
D) 7/45
62. A long-term agricultural experiment evaluates the germination of a special hybrid seed line across 10 distinct testing plots of 100 seeds each. If the experimental probabilities of failure to germinate across the 10 plots are: 0.12, 0.15, 0.08, 0.10, 0.14, 0.11, 0.09, 0.13, 0.07, and 0.11, what is the unified empirical probability that a single seed selected at random from the aggregate pool of 1000 seeds germinates successfully?
- A) 0.110
B) 0.890
C) 0.886
D) 0.114

MCQ Answer Key

One-Line Format Key: 1-C, 2-B, 3-D, 4-D, 5-B, 6-A, 7-A, 8-A, 9-B, 10-C, 11-A, 12-A, 13-B, 14-B, 15-A, 16-B, 17-B, 18-B, 19-A, 20-B.

Section B: Subjective Questions (20 Difficult Subjective Questions) Delta Level

45. An unbiased cubical die is rolled 1200 times. Let E_1 be the event of getting a factor of 6, E_2 be the event of getting a multiple of 2, and E_3 be the event of getting a perfect square. Evaluate the compound empirical probability $P(E_1 \cup E_2 \cup E_3)$ assuming the distribution aligns perfectly with classical proportions.
46. A batch of 1500 manufacturing items contains an unknown number of structural defects. A quality control tester samples 120 items at random with replacement and notes 9 defective units. When 250 more items are sampled under identical

parameters, 16 defective items are discovered. Derive the best aggregate empirical probability of selecting a non-defective unit based on the full combined dataset.

47. A coin is manipulated such that when tossed, the empirical probability of obtaining a Head over 10,000 trials is found to be 0.5004. If this coin is tossed 3 consecutive times in a simulation, list out all possible compound outcomes and explicitly calculate the experimental probability of achieving at least two consecutive Heads.
48. A group of 250 students enrolled in a senior math academy were systematically cross-tabulated according to their performance tiers in two consecutive benchmarking tests (Test Alpha and Test Beta):

Test Alpha \ Test Beta	Advanced Tier	Proficient Tier	Basic Tier
Advanced Tier	42	28	10
Proficient Tier	18	64	32
Basic Tier	5	15	36

49. Calculate the empirical probability that a student chosen entirely at random scored in the Advanced Tier in at least one of the two benchmarking tests.
50. Referring to the cross-tabulated student data in Question 4, evaluate the empirical probability that a randomly chosen student shifted their performance tier between Test Alpha and Test Beta (i.e., did not remain in the same tier).
51. A closed bag holds an assortment of 24 marbles colored either Red, White, or Crimson. A marble is drawn at random 300 times with replacement, yielding a Red marble 75 times. The marble is then modified by adding an unknown number y of Crimson marbles. If a subsequent experiment of 200 trials with the updated bag yields a White or Red marble 90 times, analytically evaluate the exact count of Crimson marbles originally present and the added value y , given that the original ratio of White to Crimson was 5:3.
52. Two separate identical cubical dice are thrown simultaneously 720 times. A student tracks the absolute difference between the numbers appearing on the top faces. Compile a structured theoretical distribution table for the absolute differences (0 through 5) and determine the compound empirical probability that the absolute difference is less than or equal to 2.
53. A localized environmental sensor records wind vectors over 365 days. The frequency of high-velocity gusts exceeding 45 km/h is recorded across the four

seasons: Spring (22 days), Summer (8 days), Autumn (14 days), and Winter (31 days). Find the empirical probability that a randomly chosen calendar day from that year fell in either Spring or Winter and did not feature a high-velocity gust. (Assume standard season lengths: Spring=92 days, Summer=92 days, Autumn=91 days, Winter=90 days).

54. An integer value x is selected at random from the set of all integers satisfying the compound inequality $-15 \leq x \leq 25$. Determine the probability that $x^2 - 2x - 8, x \leq 0$.
55. A deck of 52 cards is altered by removing all the clubs and all the red queens. If a card is selected from this depleted deck 380 times with replacement, determine the empirical frequency with which you expect to draw a black face card.
56. A computerized geometric target game registers hits within a coordinate plane. The target region is bounded by the lines $x = 0$, $x = 6$, $y = 0$, and $y = 6$. A sub-target zone is defined by the inequality $y \geq x^2$. If 1800 independent tracking points are uniformly simulated across the main square region, calculate the theoretical proportion of points expected to land inside the sub-target zone, treating this as a baseline geometric probability problem.
57. A complex manufacturing line tests 5000 integrated microcircuits. The circuits are subject to two independent quality scans: Scan A and Scan B. The log indicates that 350 circuits failed Scan A, 280 failed Scan B, and 120 circuits failed both scans simultaneously. Compute the empirical probability that a microcircuit picked at random has completely passed the quality control screening.
58. An experimental spinner has four sectors labeled 1, 2, 3, and 4. Over 2000 trials, the empirical outcomes show that sector 1 came up 450 times, sector 2 came up 550 times, and sector 3 came up 620 times. If the spinner is flipped to a dual-stage execution where it is spun twice, outline the empirical sample space and approximate the probability that the sum of two consecutive spins is exactly 5.
59. A box contains a collection of cards numbered from 100 to 999 inclusive. A card is drawn at random. Formulate an exact mathematical approach to find the probability that the three-digit number on the drawn card contains the digit 7 at least once.
60. A health clinic observes 800 patients over a year to calculate the empirical risk of catching a seasonal viral strain based on vitamin D supplementation tracking:

Supplementation Group	Contracted Virus	Did Not Contract Virus
Regular	48	352

Supplementation Group	Contracted Virus	Did Not Contract Virus
Supplementation		
No Supplementation	112	288

61. Find the absolute difference between the empirical probability of contracting the virus without supplementation versus with regular supplementation.
62. A mathematical sequence is generated by choosing an integer k at random such that $1 \leq k \leq 120$. What is the empirical probability that the term $k^3 - k$ is a multiple of 24?
63. A large urban transportation network records bus delays. Over 1000 tracked trips, a bus was delayed at the central terminal by more than 15 minutes 180 times, delayed by 5 to 15 minutes 340 times, and arrived on time or early 480 times. If two independent trips are scheduled back-to-back, calculate the compound empirical probability that at least one trip encounters a delay of more than 5 minutes.
64. A bag contains 30 distinct slips of paper containing algebraic expressions. It is found that 12 expressions are linear polynomials, 10 are quadratic polynomials, and 8 are cubic polynomials. A student draws a slip 150 times with replacement. Calculate the expected difference between the empirical frequency of drawing a linear polynomial versus a non-linear polynomial, assuming ideal distribution alignment.
65. An unweighted coin is tossed repeatedly until a Head appears or until it has been tossed 4 times. Write out the experimental sample space configurations. If this multi-stage experiment is run 1600 times, how many times do you empirically expect the experiment to terminate on exactly the 3rd toss?
66. A statistical simulation tracks the movement of a particle on a grid. The particle starts at the origin (0,0) and can move either 1 unit Right (+1,0) or 1 unit Up (0,+1) with equal probability at each step. If the particle takes 4 steps total, construct the absolute frequency landscape of all termination coordinates and calculate the probability that the particle finishes its sequence at a point whose coordinates (x,y) satisfy $x = y$.

Section A: Multiple Choice Questions (20 Tricky Olympiad MCQs) Quantum Level

63. A bag contains 90 slips numbered uniformly from 10 to 99. A student draws a slip at random. What is the empirical probability that the number N on the slip has the property that the sum of its prime factors is a perfect square?
- A) $13/90$
 - B) $7/45$
 - C) $17/90$
 - D) $8/45$
64. Three students roll a standard six-sided die one after another. They note the numbers a , b , and c . If they execute this experiment multiple times, what is the exact probability that the outcomes satisfy the strict inequality $a + b > c + 9$?
- A) $5/108$
 - B) $7/108$
 - C) $1/18$
 - D) $1/12$
65. A token is chosen at random from a collection marked with numbers from 1 to 100. What is the probability that the selected number has an odd number of positive factors and its digits sum up to a prime number?
- A) $3/100$
 - B) $1/25$
 - C) $1/50$
 - D) $7/100$
66. A coin is tossed 6 times. Let the sequence of heads (H) and tails (T) be recorded. What is the empirical probability that the sequence contains no two consecutive heads?
- A) $21/64$
 - B) $13/32$

- C) $11/32$
- D) $25/64$

67. An integer x is chosen at random such that $1 \leq x \leq 120$. What is the probability that the expression $x^3 - x^2 - x + 1$ is a multiple of 8?
- A) $1/2$
 - B) $3/4$
 - C) $5/8$
 - D) $3/8$
68. A spinner has 8 sectors numbered 1 to 8. A child spins it three times, recording the numbers as x_1, x_2, x_3 . What is the probability that $\min(x_1, x_2) = x_3$?
- A) $63/512$
 - B) $85/512$
 - C) $95/512$
 - D) $51/256$
69. Two dice are rolled. What is the probability that the product of the two face numbers is a multiple of 4 but not a multiple of 8?
- A) $5/36$
 - B) $7/36$
 - C) $1/4$
 - D) $2/9$
70. A person stands at the origin $(0,0)$ of a coordinate grid and steps 1 unit up or 1 unit right randomly with equal probability. If they make exactly 5 steps, what is the probability that they cross or touch the line $y = x + 2$?
- A) $3/16$
 - B) $5/16$
 - C) $1/4$
 - D) $7/32$
71. A box contains cards numbered 1 to 30. If three cards are drawn one by one with replacement, what is the probability that the sum of the numbers on the cards is a

multiple of 3?

- A) $1/3$
- B) $10/27$
- C) $9/28$
- D) $11/30$

72. From the set of all subsets of $S = \{1, 2, 3, 4\}$, two distinct subsets A and B are selected at random. What is the probability that the intersection of A and B is empty?

- A) $9/16$
- B) $81/256$
- C) $3/8$
- D) $1/4$

73. A clock's hour, minute, and second hands move continuously. If a random time of day is captured up to the nearest whole minute, what is the probability that the angle between the hour and minute hands is a strict obtuse angle ($90^\circ < \theta < 180^\circ$)?

- A) $1/2$
- B) $4/9$
- C) $5/12$
- D) $23/48$

74. An integer k is selected from the range $1 \leq k \leq 60$. What is the probability that the quadratic equation $x^2 - kx + 60 = 0$ has real integer roots?

- A) $1/15$
- B) $1/12$
- C) $1/10$
- D) $2/15$

75. Four vertical rods have distinct heights: 10cm, 20cm, 30cm, and 40cm. They are placed in a straight line row in a completely random order. If you look at the row from the front (the side of the first rod), a rod is visible only if all rods in front of it are shorter than it. What is the probability that exactly 2 rods are visible?

- A) $5/12$
- B) $11/24$

- C) $1/2$
- D) $3/8$

76. A box contains 50 balls numbered 1 to 50. Two balls are drawn simultaneously without replacement. What is the probability that the sum of their numbers is odd?

- A) $25/49$
- B) $1/2$
- C) $24/49$
- D) $26/51$

77. A computer generates a random sequence of 5 single-digit numbers from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ with replacement. What is the probability that the product of these 5 digits ends in the digit 5?

- A) $(5^5 - 4^5) / 10^5$
- B) $(3^5 - 2^5) / 10^5$
- C) $(5^5 - 3^5) / 10^5$
- D) $1/32$

78. A grid consists of a 3×3 square array of 9 points. Two distinct points are chosen at random. What is the probability that the line segment connecting them passes exactly through a third point of the grid?

- A) $2/9$
- B) $5/18$
- C) $7/36$
- D) $1/6$

79. A number n is randomly picked from $\{1, 2, 3, \dots, 200\}$. What is the probability that the fractional value $(n^2 + 3) / (n + 1)$ can be simplified to a whole integer?

- A) $1/100$
- B) $1/50$
- C) $3/200$
- D) $1/40$

80. Three standard dice are thrown. What is the probability that the highest number

showing on any die is exactly 4?

- A) $37/216$
- B) $27/216$
- C) $64/216$
- D) $1/6$

81. A circle is divided into 6 equal sectors marked 1, 2, 3, 4, 5, 6 clockwise. A grasshopper starts at sector 1. Every second, it flips a fair coin: if Heads, it moves 1 sector clockwise; if Tails, it moves 1 sector counter-clockwise. What is the probability that the grasshopper is back at sector 1 after exactly 4 seconds?

- A) $3/8$
- B) $5/16$
- C) $7/16$
- D) $1/4$

82. A set contains all numbers from 1 to 50. If one number x is extracted, what is the probability that x^4 leaves a remainder of 1 when divided by 5 and x is not a prime number?

- A) $14/25$
- B) $13/25$
- C) $27/50$
- D) $12/25$

MCQ Answer Key

One-Line Format Key: 1-C, 2-A, 3-B, 4-A, 5-B, 6-C, 7-B, 8-A, 9-A, 10-B, 11-B, 12-A, 13-A, 14-A, 15-A, 16-A, 17-B, 18-A, 19-A, 20-D.

Section B: Subjective Questions (20 Tricky Olympiad Subjective Questions) Quantum Level

67. A collection of cards is numbered from 1 to 1000. One card is drawn at random. Systematically count the total outcomes to find the probability that the number drawn is divisible by 4 or 6 but not divisible by 9.

68. An experiment consists of flipping a fair coin repeatedly. The process stops immediately once two heads appear in total (not necessarily consecutively) or after 5 flips have occurred. Explicitly construct the full sample space tree and find the empirical probability that the process terminates in exactly 4 flips.
69. A set A contains all integers n such that $10 \leq n \leq 100$. If an element is drawn at random from A , determine the probability that $n^2 - 1$ is a multiple of 24. Explain your number theoretic steps.
70. Three dice are thrown simultaneously 500 times. A student tracks the outcome event where the sum of the three faces is a prime number. Assuming the experimental trend aligns with the theoretical space, compute the exact fractional probability of this event by writing out all working combinations without using combinatorial formulas.
71. A regular deck of 52 playing cards is modified by removing all face cards (Kings, Queens, Jacks). From the remaining 40 cards, two cards are drawn at random without replacement. Find the probability that both cards have numbers whose product is a perfect square.
72. A line segment of length 10 cm is marked with integer coordinates from 0 to 10. Two distinct integer points a and b are selected at random. Find the probability that the distance between a and b is strictly greater than 4 cm.
73. A game features a bag containing 7 red balls and 3 blue balls. A player draws balls one by one without replacement until all 3 blue balls are extracted. Find the probability that the game ends exactly on the 6th draw by enumerating the valid sequence paths.
74. Write out the complete set of permutations for the letters in the word "M-A-T-H" using direct listing. If an arrangement is selected at random, find the probability that no vowel sits in its original natural index relative to the word "MATH".
75. A custom die has its six faces labeled with the values -3, -2, -1, 1, 2, 3. If this die is rolled three independent times, calculate the exact probability that the sum of the three resulting values equals exactly 0.
76. A bag contains tokens numbered 1 to 20. Two tokens are drawn at random without replacement. Compute the probability that the absolute difference between the two numbers is a factor of 20.
77. A regular grid map of a small city has 3 horizontal lines and 3 vertical lines forming 4 small squares. A particle starts at the bottom-left intersection and moves along the grid lines either Right or Up with equal probability at each intersection. If it must reach the top-right intersection in exactly 4 moves, map the sample tracks to find the probability that it passes through the exact center

intersection.

78. An integer z is chosen at random from $1 \leq z \leq 90$. What is the probability that the decimal representation of $1/z$ is a terminating decimal? Explicitly prove using prime factorization rules for denominators.
79. A bag contains 5 red, 4 white, and 3 black marbles. Three marbles are drawn one after another without replacement. Without using permutations or combinations formulas, list the sequence paths to find the probability that all three marbles drawn are of completely distinct colors.
80. A 3-digit number (from 100 to 999) is chosen at random. Find the exact probability that the number is a palindrome (reads the same forwards and backwards) and is also a multiple of 3.
81. A fair coin is tossed 8 times. Find the probability that the maximum number of consecutive heads achieved is exactly 2. List or group configurations carefully.
82. Two numbers x and y are chosen sequentially with replacement from the set $\{1, 2, 3, \dots, 15\}$. Find the probability that $x^2 - y^2$ is a multiple of 3.
83. A tournament matches 4 teams: A, B, C, and D. In the first round, they are paired randomly into two matches. The winners of those matches play in the final. Assuming all teams are evenly matched (each has a 50% chance of winning any given match), find the probability that team A plays against team B at some point in the tournament.
84. A target board consists of three concentric circles with radii 1 unit, 2 units, and 3 units. A dart lands randomly inside the largest circle. Calculate the geometric probability that it lands in the middle ring (between radius 1 and 2) or the inner circle, and express how this structural ratio changes if the radius of the outer circle expands to 4 units.
85. A bag holds 10 cards numbered 1 to 10. A card is drawn, its number noted, and it is replaced along with an additional duplicate card of the same value. Then a second card is drawn. Find the probability that the second card drawn shows an even number.
86. Let $S = \{1, 2, 3, \dots, 50\}$. Two distinct numbers m and n are chosen from S at random. Calculate the probability that $m \times n$ is a multiple of 5.