

# NCERT Class 9th Ganit Manjiri

## Chapter: Measuring Space: Perimeter and Area

Level: Easy | Target: Question Bank (MCQs & Subjective)

### Part A: Multiple Choice Questions (MCQs) Alpha Level

Choose the correct option for each of the following questions:

1. The perimeter of a square with a side length of 5 cm is:  
(a) 20 cm   (b) 25 cm   (c) 15 cm   (d) 10 cm
2. The area of a rectangle with a length of 6 cm and a width of 4 cm is:  
(a)  $10 \text{ cm}^2$    (b)  $20 \text{ cm}^2$    (c)  $24 \text{ cm}^2$    (d)  $12 \text{ cm}^2$
3. The area of a triangle with a base of 8 cm and a height of 5 cm is:  
(a)  $40 \text{ cm}^2$    (b)  $20 \text{ cm}^2$    (c)  $13 \text{ cm}^2$    (d)  $10 \text{ cm}^2$
4. The circumference of a circle with a radius of 7 cm is (Take  $\pi = 22/7$ ):  
(a) 44 cm   (b) 22 cm   (c) 154 cm   (d) 88 cm
5. If the area of a square is  $64 \text{ cm}^2$ , then the length of its side is:  
(a) 4 cm   (b) 8 cm   (c) 16 cm   (d) 32 cm
6. A rectangle has a length of 10 cm and a perimeter of 32 cm. Its width is:  
(a) 6 cm   (b) 12 cm   (c) 22 cm   (d) 8 cm
7. The area of a parallelogram with a base of 6 cm and a height of 4 cm is:  
(a)  $12 \text{ cm}^2$    (b)  $24 \text{ cm}^2$    (c)  $10 \text{ cm}^2$    (d)  $20 \text{ cm}^2$
8. The area of a rhombus whose diagonals are 6 cm and 8 cm is:

- (a)  $48 \text{ cm}^2$    (b)  $14 \text{ cm}^2$    (c)  $24 \text{ cm}^2$    (d)  $28 \text{ cm}^2$
9. The perimeter of a triangle with sides 3 cm, 4 cm, and 5 cm is:  
(a) 12 cm   (b) 7 cm   (c) 6 cm   (d) 9 cm
10. The perimeter of an equilateral triangle with a side of 6 cm is:  
(a) 36 cm   (b) 18 cm   (c) 12 cm   (d) 24 cm
11. The area of a circle with a radius of 14 cm is (Take  $\pi = 22/7$ ):  
(a)  $88 \text{ cm}^2$    (b)  $44 \text{ cm}^2$    (c)  $616 \text{ cm}^2$    (d)  $154 \text{ cm}^2$
12. The perimeter of a semicircle with a radius of 7 cm (including the diameter) is:  
(a) 22 cm   (b) 36 cm   (c) 44 cm   (d) 29 cm
13. If the side of a square is doubled, its area becomes:  
(a) 2 times   (b) 3 times   (c) 4 times   (d) Remains the same
14. If the length of a rectangle is doubled while its width remains the same, its area becomes:  
(a) 2 times   (b) 4 times   (c) Half   (d) Triple
15. The formula for the area of a triangle with base 'b' and height 'h' is:  
(a)  $b \times h$    (b)  $1/2 \times b \times h$    (c)  $2 \times (b + h)$    (d)  $b^2 \times h$
16. The perimeter of a regular hexagon with a side length of 'a' is:  
(a) 4a   (b) 5a   (c) 6a   (d) 8a
17. If the radius of a circle is 7 cm, its diameter is:  
(a) 3.5 cm   (b) 14 cm   (c) 21 cm   (d) 28 cm
18. The side of a square is 10 cm. Its perimeter is:

(a) 40 cm    (b) 100 cm<sup>2</sup>    (c) 20 cm    (d) 30 cm

19. The length of a rectangle is 8 cm and its diagonal is 10 cm. Its width is:

(a) 2 cm    (b) 4 cm    (c) 6 cm    (d) 5 cm

20. The area of a square whose diagonal is 10 cm long is:

(a) 100 cm<sup>2</sup>    (b) 50 cm<sup>2</sup>    (c) 25 cm<sup>2</sup>    (d) 75 cm<sup>2</sup>

## MCQ Answer Key

1-(a), 2-(c), 3-(b), 4-(a), 5-(b), 6-(a), 7-(b), 8-(c), 9-(12 cm → a), 10-(b), 11-(c), 12-(b), 13-(c), 14-(a), 15-(b), 16-(c), 17-(b), 18-(a), 19-(c), 20-(b)

## Part B: Subjective Questions (Easy) Alpha Level

Solve the following questions and show your steps clearly:

1. Find the area of a rectangle whose length is 12 cm and breadth is 8 cm.
2. Find the perimeter of a triangle with sides measuring 7 cm, 8 cm, and 9 cm.
3. A square field has a side length of 15 m. Find the cost of fencing it at \$10 per meter.
4. Find the area of a triangle with a base of 10 cm and a corresponding altitude (height) of 6 cm.
5. The circumference of a circle is 88 cm. Find its radius using  $\pi = 22/7$ .
6. The area of a rectangular plot is 440 m<sup>2</sup>. If its length is 22 m, find its breadth.
7. Find the area of a square whose perimeter is 48 cm.
8. The base of a parallelogram is twice its height. If the area is 72 cm<sup>2</sup>, find its height.

9. Find the area of a rhombus whose diagonals are of lengths 10 cm and 12 cm.
10. Find the perimeter of an equilateral triangle with a side length of 9 cm.
11. The radius of a circular pipe is 10 cm. What length of tape is required to wrap once around the pipe? (Use  $\pi = 3.14$ )
12. A table-top measures 2 m by 1 m 50 cm. What is its area in square meters?
13. Find the cost of turfing a rectangular field 50 m long and 30 m wide at the rate of \$2 per square meter.
14. If the side of a square is increased by 2 cm, by how much does its perimeter increase?
15. Find the area of a right-angled triangle whose base is 6 cm and hypotenuse is 10 cm.
16. A wire is in the shape of a square of side 10 cm. If the wire is rebent into a rectangle of length 12 cm, find its breadth.
17. Find the perimeter of a regular pentagon with each side measuring 8 cm.
18. The area of a circle is  $154 \text{ cm}^2$ . Find its diameter using  $\pi = 22/7$ .
19. Find the area of a parallelogram whose base is 8 cm and height is 5 cm.
20. A square sheet of side 4 cm is cut from a rectangular sheet of length 8 cm and breadth 6 cm. Find the area of the remaining sheet.

## Part A: Multiple Choice Questions (MCQs) Beta Level

Choose the correct option for each of the following questions:

21. If the perimeter of a semicircular protractor is 36 cm, then its diameter is (Take  $\pi = 22/7$ ):  
(a) 14 cm    (b) 7 cm    (c) 21 cm    (d) 28 cm
22. The sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. The area of the triangle is:  
(a) 432 cm<sup>2</sup>    (b) 864 cm<sup>2</sup>    (c) 576 cm<sup>2</sup>    (d) 288 cm<sup>2</sup>
23. The diagonal of a square is  $12\sqrt{2}$  cm. If its diagonal is doubled, the area of the new square will be:  
(a) 288 cm<sup>2</sup>    (b) 576 cm<sup>2</sup>    (c) 144 cm<sup>2</sup>    (d) 72 cm<sup>2</sup>
24. If the area of an equilateral triangle is  $16\sqrt{3}$  cm<sup>2</sup>, then its perimeter is:  
(a) 48 cm    (b) 24 cm    (c) 12 cm    (d) 36 cm
25. A rectangular path 2 m wide runs around the outside of a rectangular plot of 20 m by 15 m. The area of the path is:  
(a) 156 m<sup>2</sup>    (b) 140 m<sup>2</sup>    (c) 160 m<sup>2</sup>    (d) 120 m<sup>2</sup>
26. If the radius of a circle is increased by 50%, its area will increase by:  
(a) 50%    (b) 100%    (c) 125%    (d) 150%
27. The difference between the circumference and the radius of a circle is 37 cm. The area of the circle is (Take  $\pi = 22/7$ ):  
(a) 154 cm<sup>2</sup>    (b) 44 cm<sup>2</sup>    (c) 148 cm<sup>2</sup>    (d) 74 cm<sup>2</sup>
28. An isosceles right-angled triangle has an area of 200 cm<sup>2</sup>. The length of its hypotenuse is:

(a) 20 cm    (b)  $20\sqrt{2}$  cm    (c)  $10\sqrt{2}$  cm    (d) 40 cm

29. A square and an equilateral triangle have the same perimeter. If the diagonal of the square is  $6\sqrt{2}$  cm, the area of the triangle is:

(a)  $12\sqrt{3}$  cm<sup>2</sup>    (b)  $16\sqrt{3}$  cm<sup>2</sup>    (c)  $9\sqrt{3}$  cm<sup>2</sup>    (d)  $8\sqrt{3}$  cm<sup>2</sup>

30. The area of a rhombus is 96 cm<sup>2</sup> and one of its diagonals is 12 cm. The perimeter of the rhombus is:

(a) 32 cm    (b) 40 cm    (c) 48 cm    (d) 20 cm

31. The length of a rectangle is increased by 20% and its breadth is decreased by 20%. The net change in its area is:

(a) 4% increase    (b) No change    (c) 4% decrease    (d) 2% decrease

32. The adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between the longer sides is 8 cm, the distance between the shorter sides is:

(a) 12 cm    (b) 10 cm    (c) 6 cm    (d) 15 cm

33. A copper wire, when bent in the form of a square, encloses an area of 484 cm<sup>2</sup>. If the same wire is bent in the form of a circle, the area enclosed is:

(a) 616 cm<sup>2</sup>    (b) 500 cm<sup>2</sup>    (c) 484 cm<sup>2</sup>    (d) 712 cm<sup>2</sup>

34. The perimeter of a rhombus is 100 cm and one of its diagonals is 40 cm. Its area is:

(a) 500 cm<sup>2</sup>    (b) 600 cm<sup>2</sup>    (c) 400 cm<sup>2</sup>    (d) 300 cm<sup>2</sup>

35. The base of a triangle is four times its height. If its area is 50 cm<sup>2</sup>, then its base length is:

(a) 5 cm    (b) 10 cm    (c) 20 cm    (d) 15 cm

36. A square is inscribed in a circle of radius 7 cm. The area of the square is:

(a) 98 cm<sup>2</sup>    (b) 49 cm<sup>2</sup>    (c) 196 cm<sup>2</sup>    (d) 100 cm<sup>2</sup>

37. The area of a regular hexagon whose side is  $2\sqrt{3}$  cm is:  
(a)  $18\sqrt{3}$  cm<sup>2</sup>   (b)  $36\sqrt{3}$  cm<sup>2</sup>   (c)  $12\sqrt{3}$  cm<sup>2</sup>   (d)  $24\sqrt{3}$  cm<sup>2</sup>
38. The numerical values of the area and perimeter of a square are equal. The length of its diagonal is:  
(a) 4 cm   (b)  $4\sqrt{2}$  cm   (c)  $2\sqrt{2}$  cm   (d) 8 cm
39. Four equal circles, each of radius 5 cm, touch each other externally. The area of the space enclosed between them is:  
(a) 21.5 cm<sup>2</sup>   (b) 43 cm<sup>2</sup>   (c) 14.25 cm<sup>2</sup>   (d) 25 cm<sup>2</sup>
40. The area of a trapezium is 105 cm<sup>2</sup> and its height is 7 cm. If one of the parallel sides is longer than the other by 6 cm, the length of the longer side is:  
(a) 12 cm   (b) 18 cm   (c) 15 cm   (d) 21 cm

## MCQ Answer Key

1-(a), 2-(b), 3-(b), 4-(b), 5-(a), 6-(c), 7-(a), 8-(b), 9-(b), 10-(b), 11-(c), 12-(a), 13-(a), 14-(b), 15-(c), 16-(a), 17-(a), 18-(b), 19-(a), 20-(b)

## Part B: Subjective Questions (Moderate) Beta Level

Solve the following questions, ensuring all steps, formulas, and units are written down explicitly:

21. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.
22. Using Heron's formula, calculate the area of a triangular park whose sides are 50 m, 65 m, and 75 m. Also find the cost of leveling it at \$4.50 per square meter.
23. A rectangular lawn measuring 30 m by 20 m has two cross-roads, each 2 m wide, running through its center—one parallel to the length and the other parallel to the breadth. Find the area of the roads and the cost of graveling them at \$15 per square meter.

24. The perimeter of a right-angled triangle is 60 cm and its hypotenuse is 26 cm. Find the remaining two sides and calculate the area of the triangle.
25. A path of uniform width 3 m runs all around the inside of a square park of side 60 m. Find the area of the path and the cost of paving it at \$25 per square meter.
26. A racetrack is in the form of a ring whose inner circumference is 352 m and outer circumference is 396 m. Find the width of the track and its total area.
27. The perimeter of a rhombus is 52 cm and one of its diagonals is 10 cm. Find the length of the other diagonal and hence compute its area.
28. The area of a parallelogram is  $392 \text{ cm}^2$ . If its altitude is twice the corresponding base, determine both the base and the altitude.
29. A triangular board has side dimensions of 26 cm, 28 cm, and 30 cm. Find the length of the altitude drawn to the longest side.
30. The inner diameter of a circular running track is 140 m. The track is 7 m wide everywhere. Calculate the cost of fencing the outer boundary of the track at \$12 per meter.
31. An isosceles triangle has a perimeter of 32 cm and its base is 1.2 times each of the equal sides. Find the sides and the area of the triangle.
32. A rectangular sheet of paper  $44 \text{ cm} \times 18 \text{ cm}$  is rolled along its length to form an open cylinder. Find the radius of the circular cross-section formed.
33. The parallel sides of a trapezium are in the ratio 4 : 3 and the perpendicular distance between them is 12 cm. If the area of the trapezium is  $630 \text{ cm}^2$ , find the lengths of the parallel sides.

34. A square park is surrounded by a path of uniform width 2.5 m. If the area of the path is  $165 \text{ m}^2$ , find the side length of the square park.
35. Find the area of a shaded region formed inside a square of side 14 cm when four quadrants of circles of radius 7 cm are removed from its four corners.
36. The area of an equilateral triangle is  $49\sqrt{3} \text{ cm}^2$ . Taking each vertex as a center, circles are described with a radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles.
37. A design is made on a rectangular tile of dimensions  $50 \text{ cm} \times 70 \text{ cm}$ . The design consists of 8 identical triangles, each having sides 25 cm, 25 cm, and 14 cm. Find the remaining plain area of the tile.
38. The difference between the sides of a right angle in a right-angled triangle is 7 cm. If the area of the triangle is  $60 \text{ cm}^2$ , find its perimeter.
39. A floral design on a floor is made up of 16 triangular tiles. The sides of each triangular tile are 9 cm, 28 cm, and 35 cm. Find the cost of polishing the tiles at the rate of 50 paise per  $\text{cm}^2$ .
40. A chord of a circle of radius 14 cm makes a right angle at the center. Find the area of the corresponding minor segment of the circle.

## Part A: Multiple Choice Questions (MCQs) – Difficult Level Delta Level

Choose the correct option for each of the following questions:

41. The sides of a triangle are  $(x^2 + x + 1)$ ,  $(2x + 1)$ , and  $(x^2 - 1)$ . If the area of the triangle is evaluated for a valid positive integer  $x$ , the triangle will always be:  
(a) Isosceles (b) Right-angled (c) Obtuse-angled (d) Equilateral
42. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the hexagon is  $24\sqrt{3} \text{ cm}^2$ , the area of the equilateral triangle is:  
(a)  $16\sqrt{3} \text{ cm}^2$  (b)  $12\sqrt{3} \text{ cm}^2$  (c)  $24\sqrt{3} \text{ cm}^2$  (d)  $18\sqrt{3} \text{ cm}^2$
43. Three sides of a trapezium are equal to 10 cm each. For maximum area of the trapezium, the length of the fourth side must be:  
(a) 15 cm (b) 20 cm (c) 25 cm (d) 30 cm
44. Inside a square of side 'a', four semi-circles are drawn with the four sides as diameters. The area of the region common to all four semi-circles (resembling a 4-petaled flower) is:  
(a)  $(\pi/2)a^2$  (b)  $(\pi - 2)a^2$  (c)  $(\pi/2 - 1)a^2$  (d)  $(4 - \pi)/2 \times a^2$

45. If the perimeter of a right-angled triangle is  $p$  and its hypotenuse is  $h$ , then the maximum possible area of this triangle is:  
 (a)  $(p^2 - 2ph)/4$  (b)  $(p^2 + h^2 - 2ph)/8$  (c)  $h^2/2$  (d)  $(p^2 - 2ph)/8$
46. A rectangular plot is surrounded by a path of uniform width  $d$ . If the area of the path is  $A$  and its outer perimeter is  $P$ , the dimensions of the inner plot can be uniquely solved if:  
 (a)  $P^2 - 16A \geq 0$  (b)  $P^2 - 8A \geq 0$  (c)  $P^2 - 32A \geq 0$  (d)  $A^2 - 4Pd \geq 0$
47. A triangle has side lengths  $a, b, c$  such that  $(a + b + c)(a + b - c) = 3ab$ . The angle opposite to side  $c$  is:  
 (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
48. The area of a cyclic quadrilateral with successive side lengths  $a = 4$  cm,  $b = 5$  cm,  $c = 7$  cm, and  $d = 8$  cm is computed using Brahmagupta's formula. Its exact value is:  
 (a)  $12\sqrt{6}$  cm<sup>2</sup> (b)  $4\sqrt{6}$  cm<sup>2</sup> (c)  $6\sqrt{6}$  cm<sup>2</sup> (d)  $24$  cm<sup>2</sup>
49. If the area of a circle is increased by 300%, its perimeter increases by:  
 (a) 300% (b) 200% (c) 150% (d) 100%
50. In a triangle ABC, the lengths of the medians are 9 cm, 12 cm, and 15 cm. The area of the triangle ABC is:  
 (a)  $54$  cm<sup>2</sup> (b)  $72$  cm<sup>2</sup> (c)  $108$  cm<sup>2</sup> (d)  $144$  cm<sup>2</sup>
51. The perimeter of a rhombus is 40 cm and one of its acute internal angles is  $60^\circ$ . The area of the rhombus is:  
 (a)  $50\sqrt{3}$  cm<sup>2</sup> (b)  $25\sqrt{3}$  cm<sup>2</sup> (c)  $100\sqrt{3}$  cm<sup>2</sup> (d)  $75$  cm<sup>2</sup>
52. A circular wire of radius  $R$  is cut and bent into an isosceles triangle of maximum possible area. The base of this triangle is:  
 (a)  $2\pi R / 3$  (b)  $\pi R / 2$  (c)  $4\pi R / 3$  (d)  $2\pi R / 4$

53. A point P is taken inside an equilateral triangle of side 10 cm. If the sum of perpendicular distances from P to all three sides is H, then H is equal to:  
 (a) 5 cm    (b)  $5\sqrt{3}$  cm    (c)  $10\sqrt{3}$  cm    (d) Dependent on the position of P
54. The parallel sides of an isosceles trapezium are 12 cm and 20 cm. If its oblique sides are 5 cm each, the area of the trapezium is:  
 (a)  $48 \text{ cm}^2$     (b)  $64 \text{ cm}^2$     (c)  $32 \text{ cm}^2$     (d)  $56 \text{ cm}^2$
55. A sheet of paper is in the form of a sector of a circle of radius 12 cm and central angle  $120^\circ$ . If it is rolled up so that the two bounding radii touch to form a cone, the area of the base of the cone is:  
 (a)  $16\pi \text{ cm}^2$     (b)  $24\pi \text{ cm}^2$     (c)  $36\pi \text{ cm}^2$     (d)  $12\pi \text{ cm}^2$
56. The base of a triangle is enhanced by p% and its corresponding altitude is scaled down by p%. If the net change drop in area is 4%, the value of p is:  
 (a) 4    (b) 10    (c) 20    (d) 25
57. The area of a regular octagon inscribed in a circle of radius R is:  
 (a)  $2R^2\sqrt{2}$     (b)  $2R^2$     (c)  $4R^2\sqrt{2}$     (d)  $R^2(2 + \sqrt{2})$
58. An unshaded square path of uniform width runs inside a larger shaded square of side L. If the area of the path matches exactly the remaining internal square area, the width of the path is:  
 (a)  $L/2 \times (2 - \sqrt{2})$     (b)  $L/4 \times (2 - \sqrt{2})$     (c)  $L/2 \times (\sqrt{2} - 1)$     (d)  $L/4 \times (\sqrt{2} - 1)$
59. A right-angled triangle has legs a and b and hypotenuse c. A perpendicular drawn from the right angle to the hypotenuse has length p. Which of the following relations is true?  
 (a)  $p^2 = a^2 + b^2$     (b)  $1/p^2 = 1/a^2 + 1/b^2$     (c)  $p = a + b - c$     (d)  $p^2 = ab$
60. A sector of a circle with radius r has a perimeter equal to the length of the arc of a

semi-circle of same radius  $r$ . The area of the sector is:

- (a)  $\pi r^2 / 2$     (b)  $(\pi - 2)r^2 / 2$     (c)  $(\pi - 1)r^2 / 2$     (d)  $\pi r^2 / 4$

## MCQ Answer Key

1-(c), 2-(a), 3-(b), 4-(c), 5-(b), 6-(a), 7-(c), 8-(c), 9-(d), 10-(b), 11-(a), 12-(a), 13-(b), 14-(a), 15-(a), 16-(c), 17-(a), 18-(a), 19-(b), 20-(b)

## Part B: Subjective Questions – Difficult Level Delta Level

Solve the following conceptual problems by detailing proof setups, derivations, and calculations explicitly:

41. Prove that among all quadrilaterals with given perimeter  $P$ , the square yields the maximum possible enclosed area. State the explicit conditions for your geometric proof.
42. An isosceles trapezium has parallel sides of length 10 m and 26 m, and its non-parallel sides are 10 m each. A path of uniform width 1.5 m is laid along the internal boundary of the trapezium. Compute the exact remaining area inside the plot.
43. Using Heron's Formula, derive a general algebraic expression for the area of a triangle given the lengths of its three medians  $m_a$ ,  $m_b$ , and  $m_c$ .
44. A field is in the form of a quadrilateral ABCD where  $\angle B = 90^\circ$ ,  $AB = 9$  m,  $BC = 12$  m,  $CD = 25$  m, and  $AD = 20$  m. Find the exact area of the field.
45. Three circles of radius  $r = 7$  cm are placed such that each circle touches the other two externally. Calculate the exact area of the interstitial space enclosed tightly between the three boundaries. (Take  $\pi = 22/7$  and  $\sqrt{3} = 1.732$ ).
46. A rigid wire of length  $L$  is sliced into two distinct pieces. One piece is bent to form

an equilateral triangle and the other is shaped into a square. Find the ratio of the lengths of the two pieces such that the combined area of both structures is minimized.

47. In  $\triangle ABC$ , the sides are  $a$ ,  $b$ , and  $c$ . If an interior point  $O$  is at perpendicular distances  $x$ ,  $y$ ,  $z$  from sides  $a$ ,  $b$ ,  $c$  respectively, show that the minimum value of  $(x^2 + y^2 + z^2)$  is achieved when the coordinates correlate to the triangle's altitudes. Prove its mathematical relation with area  $\Delta$ .
48. A circular grass plot of radius 42 m has a gravel path running outside around it. If the area of the gravel path is equal to  $7/9$  of the area of the circular plot, determine the precise width of the path.
49. An ornament is designed in the shape of a rhombus. The sum of the lengths of its diagonals is 34 cm and its perimeter is 52 cm. Find the individual lengths of its diagonals and calculate its area.
50. A rectangle is inscribed within a semi-circle of radius  $R$  such that its base lies on the diameter. Determine the maximum possible area that this rectangle can contain in terms of  $R$ .
51. The perimeter of a triangle is  $p$  cm and its inradius is  $r$  cm. Derive the formula for its area  $\Delta = 1/2 \times p \times r$ . If  $p = 36$  cm and the area is  $54 \text{ cm}^2$ , find its inradius.
52. A regular hexagon and a regular octagon are both drawn using equal edge lengths  $s$ . Find the exact ratio of the area of the regular hexagon to that of the regular octagon. Keep your expression radicalized.
53. A farmer has a triangular field with sides 30 m, 40 m, and 50 m. He wishes to partition the field into two plots of equal area by running a straight fence parallel to the longest side. Calculate the precise distance of this fence from the vertex opposite to the longest side.

54. Inside an equilateral triangle of side  $a$ , three equal circles are drawn such that each circle touches the other two circles and also touches two sides of the triangle. Show that the radius of each circle is given by  $r = a / [2(1 + \sqrt{3})]$ .
55. A trapezium ABCD has parallel sides  $AB \parallel CD$ , where  $AB = 40$  cm and  $CD = 24$  cm. The oblique sides are 17 cm and 15 cm. Find the height of the trapezium and its total area.
56. A sector of a circle with central angle  $\theta$  and radius  $R$  is modified. Its central angle is increased by 20% while its radius is shortened by 10%. Find the net percentage variation in the area of the sector.
57. Four matching coins of radius 2 cm are positioned on a square grid profile. A small square is placed perfectly in the center space left between the four coins such that it touches all four coins externally. Calculate the perimeter and area of this small central square.
58. In a right-angled triangle ABC, right-angled at B, a circle is inscribed touching all three sides. If the sides containing the right angle are  $a$  and  $b$ , prove that the diameter of the incircle is  $d = a + b - c$ , where  $c$  is the hypotenuse.
59. A plot of land is bounded by a parabolic curved stream profile and a straight highway line, creating an approximate non-standard triangle. The surveyor uses a multi-stage approximation with trapeziums of widths 5 m each. If the successive offsets measured are 0, 4, 7, 9, 8, 5, and 0 m, find the estimated total area using Simpson's Rule.
60. Find the area of a cyclic quadrilateral whose sides are  $a = 10$  cm,  $b = 24$  cm,  $c = 16$  cm, and the diagonal  $d_1 = 26$  cm divides it into two distinct triangles. Prove whether the quadrilateral forms a right angle inside its vertices.

## Part A: Multiple Choice Questions (MCQs) – Olympiad Level Quantum Level

Choose the correct option for each of the following questions. All problems are bounded strictly within the geometry of Class 9 (Heron's formula, lines, triangles, quadrilaterals, circles) but require non-trivial logic, parity, or optimization:

61. A triangle has integer side lengths values  $a$ ,  $b$ , and  $c$  such that  $a \times b \times c = 60$ . How many distinct non-congruent triangles can be formed?  
(a) 4    (b) 5    (c) 6    (d) 7
62. An equilateral triangle ABC has an area of  $12 \text{ cm}^2$ . Points D, E, and F are chosen on sides BC, CA, and AB respectively such that  $BD = 2CD$ ,  $CE = 2AE$ , and  $AF = 2BF$ . The area of the inner triangle formed by the intersections of the lines AD, BE, and CF is:  
(a)  $1.2 \text{ cm}^2$     (b)  $1.5 \text{ cm}^2$     (c)  $1.71 \text{ cm}^2$     (d)  $2 \text{ cm}^2$
63. In a triangle with sides 13, 14, and 15, a straight cut parallel to the side of length 14 divides the triangle into a smaller triangle and a trapezium such that their perimeters are exactly equal. The length of the cut is:  
(a) 7    (b) 8    (c) 9.8    (d) 10.5
64. A rectangular floor of dimension 14 cm by 22 cm is to be tiled completely using non-overlapping tiles shaped like a right-angled triangle with side lengths 3 cm, 4 cm, and 5 cm. What is the minimum number of tiles that must be broken or cut if the tiles cannot exit the boundary?  
(a) 0    (b) 2    (c) 4    (d) It is impossible to tile without cuts
65. A convex quadrilateral ABCD has perpendicular diagonals AC and BD. If  $AB = 13$ ,  $BC = 20$ , and  $CD = 21$ , then the exact length of side AD is:  
(a) 11    (b) 12    (c) 14    (d) 15

66. A point P inside an equilateral triangle has perpendicular distances of 3 cm, 4 cm, and 5 cm to the three sides. The total area of this equilateral triangle is:  
 (a)  $48\sqrt{3} \text{ cm}^2$  (b)  $36\sqrt{3} \text{ cm}^2$  (c)  $24\sqrt{3} \text{ cm}^2$  (d)  $16\sqrt{3} \text{ cm}^2$
67. The perimeter of a right-angled triangle with integer sides is 56 cm. What is the maximum possible area of this triangle?  
 (a)  $84 \text{ cm}^2$  (b)  $98 \text{ cm}^2$  (c)  $120 \text{ cm}^2$  (d)  $112 \text{ cm}^2$
68. A square sheet of side 10 cm has four identical symmetrical triangular flaps cut out from its corners to reshape it into a regular octagon. The side length of this regular octagon is:  
 (a)  $10\sqrt{2} - 10 \text{ cm}$  (b)  $10 / (\sqrt{2} + 1) \text{ cm}$  (c)  $10(\sqrt{2} - 1) \text{ cm}$  (d)  $5\sqrt{2} \text{ cm}$
69. A circle is inscribed inside a right-angled triangle whose legs are 8 cm and 15 cm. The area of the circle is:  
 (a)  $4\pi \text{ cm}^2$  (b)  $9\pi \text{ cm}^2$  (c)  $16\pi \text{ cm}^2$  (d)  $2.25\pi \text{ cm}^2$
70. In  $\triangle ABC$ , the median AD is perpendicular to the median BE. If AC = 15 cm and BC = 9 cm, then the length of AB is:  
 (a)  $3\sqrt{5} \text{ cm}$  (b) 6 cm (c)  $\sqrt{34} \text{ cm}$  (d)  $\sqrt{61} \text{ cm}$
71. A wire frame in the shape of an equilateral triangle encloses a maximum area A. If the same wire is cut into two equal pieces, and each piece is bent into a smaller square, the sum of the areas of the two squares is related to A by:  
 (a)  $3A / 16$  (b)  $9A / (32\sqrt{3})$  (c)  $3\sqrt{3} \times A / 32$  (d)  $3A / (8\sqrt{3})$
72. The sides of a triangle are consecutive integers, and its area is also an integer. Which of the following cannot be the perimeter of such a triangle?  
 (a) 12 (b) 42 (c) 60 (d) 174
73. A regular hexagon is inscribed in a circle of radius R, and another regular hexagon circumscribes the same circle. The ratio of the area of the inscribed hexagon to the circumscribed hexagon is:

- (a)  $3 : 4$    (b)  $2 : 3$    (c)  $\sqrt{3} : 2$    (d)  $1 : \sqrt{2}$

74. A rectangle ABCD has an area of  $48 \text{ cm}^2$ . A point P is taken anywhere inside the rectangle. The sum of the areas of  $\triangle PAB$  and  $\triangle PCD$  is:  
(a)  $12 \text{ cm}^2$    (b)  $16 \text{ cm}^2$    (c)  $24 \text{ cm}^2$    (d) Changes depending on P
75. Four circles of radius 1 cm are arranged in a straight row such that each circle is tangent to its neighbor(s). A tight string is wrapped around this entire assembly. The length of the string is:  
(a)  $12 + 2\pi \text{ cm}$    (b)  $6 + 2\pi \text{ cm}$    (c)  $12 + \pi \text{ cm}$    (d)  $8 + 2\pi \text{ cm}$
76. Two sides of a triangle have lengths 10 cm and 15 cm. If the area of the triangle is a maximum, the length of the third side must be:  
(a)  $5\sqrt{13} \text{ cm}$    (b) 25 cm   (c)  $5\sqrt{5} \text{ cm}$    (d) 18 cm
77. The perimeter of a kite is 60 cm, and its diagonals are 12 cm and 16 cm long. The area of the kite is:  
(a)  $192 \text{ cm}^2$    (b)  $96 \text{ cm}^2$    (c)  $120 \text{ cm}^2$    (d)  $150 \text{ cm}^2$
78. A trapezium ABCD has parallel sides  $AB = 10 \text{ cm}$  and  $CD = 25 \text{ cm}$ . If the area of  $\triangle AOB$  (where AB is base and top vertex is intersection of diagonals O) is  $4 \text{ cm}^2$ , the total area of the trapezium is:  
(a)  $36 \text{ cm}^2$    (b)  $49 \text{ cm}^2$    (c)  $50 \text{ cm}^2$    (d)  $42.25 \text{ cm}^2$
79. A square ABCD of side length 2 cm contains a smaller shaded region. This region is bounded by two circular arcs: one drawn with center A and radius 2 cm, and another drawn with center C and radius 2 cm. The area of this lens-shaped region is:  
(a)  $2\pi - 4 \text{ cm}^2$    (b)  $\pi - 2 \text{ cm}^2$    (c)  $4\pi - 8 \text{ cm}^2$    (d)  $2\pi - 2 \text{ cm}^2$
80. If the sides of a triangle are in the ratio  $4 : 5 : 6$ , the ratio of its longest altitude to its shortest altitude is:  
(a)  $6 : 4$    (b)  $5 : 4$    (c)  $3 : 2$    (d)  $5 : 3$

## MCQ Answer Key

1-(a), 2-(c), 3-(c), 4(--a), 5-(c), 6-(a), 7-(a), 8-(c), 9-(b), 10-(a), 11-(c), 12-(c), 13-(a), 14-(c), 15-(a), 16-(a), 17-(b), 18-(b), 19-(a), 20-(a)

## Part B: Subjective Questions – Olympiad Level

### Quantum Level

Solve the following problems by applying standard Class 9 geometric concepts (Heron's formula, mid-point theorem, areas of parallelograms, circle properties) combined with advanced mathematical logic:

61. In  $\triangle ABC$ , the sides are  $BC = a$ ,  $AC = b$ , and  $AB = c$ . A straight line is drawn parallel to  $BC$  that cuts  $AB$  at  $P$  and  $AC$  at  $Q$ . If the line segment  $PQ$  divides the triangle into two regions (a smaller triangle and a trapezium) of equal area, prove that the perimeter of  $\triangle APQ$  is strictly less than the perimeter of the trapezium  $BPQC$ . Find the exact numerical ratio of their perimeters.
62. A rectangular plot of dimensions 30 m by 20 m has a walking path running around its internal perimeter. The area of the path is exactly equal to the area of the remaining inner rectangle. Find the width of the path up to two decimal places without using the quadratic formula formula components exceeding class 9 scope (solve using factorization).
63. A triangle has side lengths of 25 cm, 39 cm, and 56 cm. Calculate the area of the triangle using Heron's formula. Also, find the length of the perpendicular dropped from the vertex opposite to the side of length 56 cm.
64. A parallelogram  $ABCD$  has adjacent sides of lengths 12 cm and 16 cm. If the distance between the longer parallel sides is 6 cm, find the distance between the shorter parallel sides. Show that the ratio of the distances between parallel lines is inversely proportional to the ratio of their lengths.
65. A convex quadrilateral  $ABCD$  has side lengths  $AB = 3$  cm,  $BC = 4$  cm,  $CD = 12$  cm,

and  $DA = 13$  cm. If  $\angle ABC = 90^\circ$ , calculate the exact area of the quadrilateral without assuming it is cyclic.

66. An equilateral triangle is constructed on one side of a square of side ' $s$ ' externally. Find the total area and perimeter of the combined pentagonal figure in terms of ' $s$ '.
67. A circle is inscribed inside an isosceles triangle whose equal sides are 10 cm each and base is 12 cm. Find the radius of this incircle using the relationship between area and semi-perimeter.
68. A trapezium ABCD with  $AB \parallel CD$  has diagonals AC and BD intersecting at point O. Prove that the area of  $\triangle AOD$  is exactly equal to the area of  $\triangle BOC$ , regardless of the non-parallel side configurations.
69. A regular hexagon ABCDEF has an area of  $72 \text{ cm}^2$ . Points P, Q, and R are the midpoints of alternative sides AB, CD, and EF respectively. Calculate the area of the triangle PQR formed by joining these midpoints.
70. The perimeter of a triangle is 32 cm. One side is 11 cm and the difference between the other two sides is 5 cm. Find the area of the triangle using Heron's formula.
71. A wire of length 48 cm is cut into two pieces. One piece is bent to form a square, and the other is bent to form an equilateral triangle. If the area of the square is exactly 3 times the area of the equilateral triangle, find the lengths of the two wire pieces.
72. A rectangular sheet of paper has dimensions  $32 \text{ cm} \times 20 \text{ cm}$ . Four congruent squares of side  $x$  cm are cut out from the four corners, and the remaining paper is folded up to form an open box. If the sum of the areas of the base and the four vertical sides is  $524 \text{ cm}^2$ , calculate the value of  $x$ .
73. A point O inside a rectangle ABCD is joined to all four vertices. Prove that the sum

of the areas of opposite triangles is equal, i.e.,  $\text{Area}(\triangle OAB) + \text{Area}(\triangle OCD) = \text{Area}(\triangle OBC) + \text{Area}(\triangle OAD)$ .

74. The base of an isosceles triangle is 24 cm and its area is  $192 \text{ cm}^2$ . Find its perimeter and the length of the altitude drawn to one of the equal sides.
75. A field is in the form of a rhombus whose perimeter is 80 m and one of its diagonals is 24 m. Find the cost of leveling the field at \$3.50 per square meter.
76. Three identical circles of radius 14 cm are placed on a table such that each circle touches the other two externally. Find the area of the region enclosed between the three circles. (Take  $\pi = 22/7$ ,  $\sqrt{3} = 1.732$ ).
77. A path of uniform width  $x$  runs around the outside of a circular park of radius 7 m. If the area of the path is equal to the area of the park, find the exact width  $x$  of the path in radical form.
78. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm, and 30 cm, and the parallelogram stands on the base 28 cm, find the vertical height of the parallelogram.
79. A square ABCD is inscribed inside a circle of radius 7 cm. Find the total area of the four segments of the circle lying outside the square. (Take  $\pi = 22/7$ ).
80. In a right-angled triangle ABC, with  $\angle B = 90^\circ$ , a perpendicular BD is dropped onto the hypotenuse AC. If the area of  $\triangle ABC$  is  $60 \text{ cm}^2$  and  $AC = 13 \text{ cm}$ , find the length of BD and the individual areas of  $\triangle ABD$  and  $\triangle CBD$ .