

NCERT Class 9 Mathematics (Ganit Manjari)

Chapter-5 I'm Up and Down, and Round and Round

Alpha MCQ

Q1. The set of all points in a plane which are at a fixed distance from a fixed point is called a:

- (A) Square
- (B) Circle
- (C) Triangle
- (D) Sector

Q2. The longest chord of a circle is its:

- (A) Radius
- (B) Minor arc
- (C) Diameter
- (D) Segment

Q3. How many lines of reflection symmetry does a circle have?

- (A) One
- (B) Two
- (C) Four
- (D) Infinitely many

Q4. A unique circle can be drawn through how many non-collinear points?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Q5. If a circle has a radius of 7 cm, what is the length of its longest chord?

- (A) 7 cm
- (B) 14 cm
- (C) 3.5 cm

(D) 21 cm

Q6. The points that all lie on the same circle are called:

(A) Collinear

(B) Coplanar

(C) Concyclic

(D) Concurrent

Q7. For a right-angled triangle, the circumcentre always lies:

(A) Inside the triangle

(B) Outside the triangle

(C) At the midpoint of the hypotenuse

(D) At the vertex containing the right angle

Q8. The angle subtended by a diameter at any point on the remaining part of the circle is:

(A) 45°

(B) 60°

(C) 90°

(D) 180°

Q9. If the opposite angles of a quadrilateral add up to 180° , the quadrilateral must be:

(A) A parallelogram

(B) A cyclic quadrilateral

(C) A rhombus

(D) A trapezium

Q10. The perpendicular dropped from the centre of a circle to a chord always:

(A) Bisects the chord

(B) Triples the chord

(C) Is parallel to the chord

(D) Doubles the chord length

Q11. Chords of a circle which are equidistant from the centre are always:

(A) Perpendicular to each other

(B) Equal in length

(C) Intersecting at right angles

(D) Different in length

Q12. If an arc subtends an angle of 80° at the centre of a circle, the angle subtended by it at any point on the remaining part of the circle is:

(A) 80°

(B) 160°

(C) 40°

(D) 20°

Q13. In a cyclic quadrilateral ABCD, if angle A = 70° , then the measure of angle C is:

(A) 70°

(B) 110°

(C) 90°

(D) 180°

Q14. How many circles can pass through two fixed points?

(A) Only one

(B) Exactly two

(C) None

(D) Infinitely many

Q15. The rotational symmetry of a circle about its centre is:

(A) Zero

(B) Limited to 4 angles

(C) Infinite

(D) Limited to 2 angles

Q16. For an obtuse-angled triangle, the circumcentre lies:

(A) Inside the triangle

(B) Outside the triangle

(C) On the longest side

(D) At one of the vertices

Q17. A parallelogram inscribed inside a circle must be a:

(A) Rhombus

(B) Trapezium

(C) Rectangle

(D) Kite

Q18. If the central angle subtended by a chord AB is 60° and the radius of the circle is 5 cm, the length of chord AB is:

(A) 5 cm

(B) 10 cm

(C) 2.5 cm

(D) 7.5 cm

Q19. Every line of reflection symmetry in a circle must pass through its:

(A) Boundary

(B) Segment

(C) Minor arc

(D) Centre

Q20. If two chords of a circle subtend equal angles at the centre, then the chords are:

(A) Parallel

(B) Perpendicular

(C) Equal in length

(D) Unequal in length

MCQ Answer Key

1. B

2. C

3. D

4. C

5. B

6. C

7. C

8. C

9. B

10. A

11. B

12. C

13. B

14. D 15. C

16. B

17. C

18. A

19. D

20. C

Alpha Subjective

Q1. Define a chord of a circle.

Q2. What is the relationship between the radius and the diameter of a circle?

Q3. Where does the circumcentre of an acute-angled triangle lie?

Q4. A chord of length 24 cm is drawn in a circle of radius 13 cm. Find its distance from the centre.

Q5. What is the sum of the opposite angles of a cyclic quadrilateral?

Q6. An arc subtends an angle of 50° at a point on the circle. What is the angle subtended by this arc at the centre?

Q7. If the distance of a chord from the centre of a circle is 0 cm, what special name is given to that chord?

Q8. How many circles can be drawn passing through three collinear points?

Q9. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, what can you say about these four points?

Q10. What is the name of a circle that passes through all three vertices of a triangle?

Q11. If two circles intersect at two points, where do their centres lie relative to the common chord?

Q12. If a chord of length 10 cm is at a distance of 4 cm from the centre, what will be the length of another chord that is also 4 cm away from the centre?

Q13. What is the least possible radius of a circle passing through two fixed points A and B that are 10 cm apart?

- Q14. In a cyclic quadrilateral PQRS, if angle P = angle R, what is the measure of angle P?
- Q15. The line joining the centre of a circle to the midpoint of a chord forms what angle with the chord?
- Q16. What is a continuous, connected piece of a circle's boundary called?
- Q17. If a circle is folded perfectly in half, what geometric feature does the crease represent?
- Q18. If the radius of a circumcircle of a right-angled triangle is 6 cm, what is the length of the triangle's hypotenuse?
- Q19. An exterior angle of a cyclic quadrilateral is equal to which interior angle?
- Q20. If a triangle AOB is formed by two radii OA and OB and a chord AB in a circle, what type of triangle is AOB always guaranteed to be at minimum?

Subjective Answer Key

1. A line segment joining any two distinct points on a circle.
2. The diameter is twice the length of the radius (Diameter = 2 x radius).
3. Inside the triangle.
4. 5 cm
5. 180°
6. 100°
7. Diameter
8. Zero (No circle can pass through three collinear points).
9. They are concyclic (they lie on the same circle).
10. Circumcircle
11. On the perpendicular bisector of the common chord.
12. 10 cm
13. 5 cm
14. 90°
15. 90° (It is perpendicular to the chord).
16. An arc
17. A diameter (or line of symmetry).
18. 12 cm
19. The interior opposite angle.
20. An isosceles triangle.

Beta MCQ

1. The longest chord of a circle is called:
 - A. Radius
 - B. Secant
 - C. Diameter
 - D. Tangent
2. If the distance of a chord from the center of a circle increases, the length of the chord:
 - A. Increases
 - B. Decreases

- C. Remains same
 - D. Becomes zero
3. The angle subtended by a diameter at any point on the semi-circle is:
- A. 45 degrees
 - B. 60 degrees
 - C. 90 degrees
 - D. 180 degrees
4. How many circles can pass through three non-collinear points?
- A. Only one
 - B. Two
 - C. Three
 - D. Infinite
5. In a circle, if chord $AB = 10$ cm and chord $CD = 10$ cm, and the distance of AB from the center is 4 cm, then the distance of CD from the center is:
- A. 5 cm
 - B. 10 cm
 - C. 4 cm
 - D. 8 cm
6. The sum of opposite angles of a cyclic quadrilateral is:
- A. 90 degrees
 - B. 180 degrees
 - C. 270 degrees
 - D. 360 degrees
7. A chord of length 16 cm is drawn in a circle of radius 10 cm. Its distance from the center is:
- A. 8 cm
 - B. 6 cm
 - C. 4 cm
 - D. 12 cm
8. If the degree measure of an arc is 120 degrees, the angle subtended by it at the center is:
- A. 60 degrees
 - B. 120 degrees
 - C. 240 degrees
 - D. 90 degrees
9. Angles in the same segment of a circle are:
- A. Complementary

- B. Supplementary
 - C. Equal
 - D. Unequal
10. [Diagram Question] In a given circle with center O, a cyclic quadrilateral ABCD is drawn. If Angle A = 75 degrees, then the measure of its opposite Angle C is:
- A. 75 degrees
 - B. 105 degrees
 - C. 180 degrees
 - D. 15 degrees
11. If the length of a chord is equal to the radius of the circle, the angle subtended by it at the center is:
- A. 30 degrees
 - B. 45 degrees
 - C. 60 degrees
 - D. 90 degrees
12. The region between an arc and the two radii joining the center to the endpoints of the arc is:
- A. Segment
 - B. Sector
 - C. Chord
 - D. Circumference
13. [Diagram Question] In a figure showing a circle with center O, points B and C lie on the circumference such that Angle BOC = 110 degrees. Point A lies on the major arc BC. Find the value of Angle BAC:
- A. 55 degrees
 - B. 110 degrees
 - C. 220 degrees
 - D. 70 degrees
14. Two circles are congruent if they have the same:
- A. Center
 - B. Chord length
 - C. Radius
 - D. Segment
15. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are:
- A. Collinear

- B. Concyclic
 - C. Non-coplanar
 - D. Vertices of a square
16. [Diagram Question] In a circle illustrating a cyclic quadrilateral ABCD, if Angle A - Angle C = 20 degrees, then the measure of Angle A is:
- A. 100 degrees
 - B. 80 degrees
 - C. 90 degrees
 - D. 110 degrees
17. The perpendicular from the center of a circle to a chord:
- A. Trisects the chord
 - B. Bisects the chord
 - C. Is parallel to the chord
 - D. Is double the chord
18. [Diagram Question] In a figure, a chord AB is drawn in a circle with center O. If chord AB subtends 60 degrees at the center, then the triangle OAB is:
- A. Right angled
 - B. Isosceles but not equilateral
 - C. Equilateral
 - D. Scalene
19. Number of tangents that can be drawn from a point inside the circle is:
- A. 0
 - B. 1
 - C. 2
 - D. Infinite
20. A circle divides the plane on which it lies into how many parts?
- A. 1
 - B. 2
 - C. 3
 - D. 4

Beta Subjective

1. [Diagram Question] A circle is drawn with center O and radius 13 cm. A perpendicular line from O meets a chord AB at point M. If OM = 5 cm, find the length of the chord AB.
2. In a cyclic quadrilateral ABCD, if Angle B = 85 degrees and Angle D = $(x + 5)$ degrees, find the value of x.

3. If the radius of a circle is 5 cm, find the length of the longest chord.
4. AB and CD are two equal chords of a circle with center O. If Angle AOB = 70 degrees, find Angle COD.
5. An arc of a circle is $\frac{1}{4}$ of the circumference. Find the angle subtended by the arc at the center.
6. If the measure of an arc is 80 degrees, find the angle subtended by it at any point on the remaining part of the circle.
7. [Diagram Question] In a circle of radius 17 cm, a chord AB of length 30 cm is drawn. A perpendicular line segment OM is drawn from the center O to the chord AB. Find the length of OM.
8. Two chords AB and AC of a circle are equal. Prove that the center of the circle lies on the bisector of Angle BAC. (State the final conclusion).
9. [Diagram Question] Consider a circle with center O. Points P, Q, and R lie on the circumference such that Angle PQR = 100 degrees. Find the measure of reflex Angle POR.
10. If the non-parallel sides of a trapezium are equal, what kind of quadrilateral is it?
11. [Diagram Question] In a circle with center O, triangle OAB is formed where A and B lie on the circumference. If a point C on the major arc forms Angle ACB = 40 degrees, find Angle OAB.
12. Find the radius of a circle if a chord of length 24 cm is at a distance of 5 cm from the center.
13. If two equal chords of a circle intersect within the circle, what can you say about the segments of one chord relative to the other?
14. A circle has a radius of 7 cm. What is the distance between two parallel tangents?
15. [Diagram Question] In a circle with center O, radii OP and OR are drawn. If Angle OPR = 30 degrees, find Angle POR.
16. Find the sum of Angle A + Angle C in a cyclic quadrilateral ABCD.
17. If a chord AB of a circle subtends Angle ACB = 30 degrees on the circumference, find the angle it subtends at the center.
18. [Diagram Question] In a circle with radius 5 cm, two parallel chords AB = 6 cm and CD = 8 cm are drawn on the same side of the center. Find the distance between these two chords.
19. ABCD is a cyclic quadrilateral. If Angle A = $2x + 4$ and Angle C = $4x - 4$, find x.
20. What is the length of a chord that subtends 90 degrees at the center of a

circle with radius r ?

Answer Key

Section A: MCQs

1. C | 2. B | 3. C | 4. A | 5. C | 6. B | 7. B | 8. B | 9. C | 10. B | 11. C | 12. B | 13. A | 14. C | 15. B | 16. A | 17. B | 18. C | 19. A | 20. C

Section B: Subjective

1. 24 cm 2. 90 3. 10 cm 4. 70 degrees 5. 90 degrees
6. 40 degrees 7. 8 cm 8. Center lies on the angle bisector. 9. 200 degrees
10. Cyclic Trapezium 11. 50 degrees 12. 13 cm 13. Segments are equal.
14. 14 cm 15. 120 degrees 16. 180 degrees 17. 60 degrees
18. 1 cm 19. 30 20. $r \times \text{square root of } 2$

Delta MCQ

- Two parallel chords of a circle are 10 cm and 24 cm long. If the radius of the circle is 13 cm, the distance between the two chords when they lie on opposite sides of the center is:
A. 7 cm B. 12 cm C. 17 cm D. 19 cm
- If a cyclic quadrilateral ABCD has its diagonals intersecting at right angles at a point P, then the line drawn from P perpendicular to AB, when extended backwards, will always:
A. Bisect the opposite side CD B. Be perpendicular to CD
C. Pass through the center O D. Trisect the chord CD
- In a circle, a chord AB subtends an angle of 60 degrees at the center O. If a variable point P moves on the minor arc AB (excluding A and B), then the measure of Angle APB is always:
A. 30 degrees B. 120 degrees
C. 150 degrees D. Variable depending on the exact position of P
- An equilateral triangle ABC is inscribed in a circle with center O. A point P lies on the minor arc BC. The relationship between the lengths of segments PA, PB, and PC is given by:

- A. $PA = PB + PC$ B. $PA^2 = PB^2 + PC^2$
 C. $PA = \sqrt{PB \cdot PC}$ D. $PB = PA + PC$
5. In a circle of radius R , two chords AB and CD intersect orthogonally at an internal point P . The value of $(AB^2 + CD^2)$ cannot be determined, but the sum of the squares of the segments $(AP^2 + BP^2 + CP^2 + DP^2)$ is always equal to:
- A. $2 \cdot R^2$ B. $4 \cdot R^2$ C. $3 \cdot R^2$ D. R^2
6. ABC is a triangle inscribed in a circle. The internal bisectors of Angle A , Angle B , and Angle C intersect the circumcircle at points D , E , and F respectively. The angles of triangle DEF will be:
- A. $90 - A/2, 90 - B/2, 90 - C/2$ B. $A/2, B/2, C/2$
 C. $180 - 2A, 180 - 2B, 180 - 2C$ D. $60, 60, 60$
7. Let $ABCD$ be a cyclic quadrilateral such that AB is the diameter of the circle. If Angle $ADC = 140$ degrees, then the measure of Angle BAC is:
- A. 40 degrees B. 50 degrees C. 60 degrees D. 30 degrees
8. Three congruent circles, each of radius r , touch each other externally. The radius of the smallest circle that can enclose all three touching circles simultaneously is:
- A. $2r$ B. $r \cdot (1 + 2 / \sqrt{3})$
 C. $r \cdot (2 + \sqrt{3})$ D. $3r$
9. If the length of a chord of a circle is equal to its distance from the center, then the radius of the circle in terms of the chord length L is:
- A. $L \cdot \sqrt{5/2}$ B. $L / \sqrt{2}$
 C. $L \cdot \sqrt{3/2}$ D. $L \cdot \sqrt{2}$
10. Through a fixed point P inside a circle of radius 5 cm, the shortest chord that can be drawn is of length 6 cm. The distance of the point P from the center of the circle is:
- A. 3 cm B. 4 cm C. 2 cm D. 4.5 cm
11. $ABCD$ is a cyclic quadrilateral with extended sides AB and DC intersecting at P , and extended sides AD and BC intersecting at Q . If the bisectors of Angle P and Angle Q meet at point R , the angle between the two bisectors at R is:
- A. 90 degrees B. 45 degrees
 C. 60 degrees D. 0 degrees (They are parallel)
12. Two circles intersect at points P and Q . A straight line through P intersects the first circle at A and the second circle at B . If the tangents to the circles at A and B intersect at T , then the quadrilateral $TAQB$ is:

- A. Cyclic B. Trapezium C. Parallelogram D. Rhombus
13. In a circle with center O, chords AB and CD are equal and intersect at a point E inside the circle. Which of the following statements is false?
 A. $AE = CE$ B. $BE = DE$
 C. OE bisects Angle AED D. Triangle AEC is always equilateral
14. The locus of the midpoints of all chords of a circle that pass through a fixed point P inside the circle forms:
 A. Another concentric circle B. A straight line segment
 C. A circle with OP as the diameter D. A parabola
15. If a line intersects two concentric circles with center O at points A, B, C, and D in that specific order along the line, then AB is always equal to:
 A. CD B. BC
 C. $AD / 2$ D. $AC - BD$
16. In a cyclic quadrilateral ABCD, the perpendicular bisectors of the sides AB, BC, CD, and DA:
 A. Form a smaller cyclic quadrilateral B. Intersect at a single concurrent point
 C. Are parallel in pairs D. Do not intersect
17. If a regular octagon is inscribed in a circle of radius R, the length of one side of the octagon is:
 A. $R * \sqrt{2 - \sqrt{2}}$ B. $R * \sqrt{2}$
 C. $R * (\sqrt{2} - 1)$ D. $R / \sqrt{2}$
18. A circle of radius 2 cm rolls around the outside of a fixed circle of radius 6 cm without slipping until it returns to its starting point. How many full rotations does the smaller circle make around its own axis?
 A. 3 B. 4 C. 2 D. 5
19. If the ratio of the areas of a regular hexagon inscribed in a circle to that of a regular hexagon circumscribed about the same circle is 3:4, what is the ratio of their perimeters?
 A. 3:4 B. square root of 3 : 2 C. 9:16 D. 1 : square root of 2
20. A chord AB divides a circle into two segments. If a point C is taken on the minor arc and a point D on the major arc, then the value of (Angle ACB - Angle ADB) is:
 A. 90 degrees B. $180 \text{ degrees} - 2 * \text{Angle ADB}$
 C. Always constant at 0 degrees D. Dependent on the length of the radius

Delta Subjective

1. A point P is at a distance of 10 cm from the center of a circle of radius 6 cm. Find the length of the shortest chord passing through P.
2. In a cyclic quadrilateral ABCD, side AB is produced to E. If Angle CBE = 108 degrees, find the measure of its interior opposite Angle ADC.
3. Two parallel chords of a circle have lengths 12 cm and 16 cm. If the distance between them is 2 cm and they lie on the same side of the center, calculate the radius of the circle.
4. An isosceles triangle ABC with $AB = AC$ is inscribed in a circle. If Angle A = 50 degrees, determine the angle subtended by the minor arc BC at the center of the circle.
5. In a circle with center O, chords AB and CD intersect at an interior point E. If Angle AEC = 45 degrees, find the sum of the angles subtended by arcs AC and BD at the center.
6. A chord of length 14 cm is at a distance of 24 cm from the center of a circle. Find the length of another chord of the same circle which is at a distance of 7 cm from the center.
7. If the diagonals of a cyclic quadrilateral ABCD pass through the center O of the circle, identify the specific mathematical name of the quadrilateral ABCD.
8. Two circles intersect at points A and B. Through A, two lines CAD and EAF are drawn intersecting the circles at C, D and E, F respectively. If CE = 5 cm, find the length of DF.
9. In a circle, a chord AB of length 8 cm is at a distance of 3 cm from the center O. Find the distance from O to a chord CD of length 6 cm in the same circle.
10. If the sum of a pair of opposite angles of a quadrilateral is 180 degrees, where do all four vertices of the quadrilateral lie?
11. A triangle ABC is inscribed in a circle with center O. If Angle OBC = 25 degrees, find the measure of Angle BAC.
12. If a cyclic trapezium has its non-parallel sides equal to 5 cm each and its parallel bases are 10 cm and 16 cm, find the distance between its parallel sides.
13. In a circle, a chord AB subtends an angle of 90 degrees at the center. Find the ratio of the length of the chord to the radius of the circle.
14. If a circle is inscribed inside a right-angled triangle with sides 6 cm, 8 cm,

and 10 cm, find the radius of the circle.

15. Two equal chords AB and CD of a circle with center O intersect at right angles at point E. If the radius of the circle is R, find the length of OE in terms of the segments AE and BE.
16. Find the angle subtended at the circumference by an arc whose length is exactly equal to one-sixth of the total circumference of the circle.
17. A circle passes through the vertices A, B, and C of a parallelogram ABCD. If the circle intersects the extended line CD at point P, find the geometric relationship between sides AB and AP.
18. In a cyclic quadrilateral ABCD, Angle A is thrice Angle C, and Angle B is five times Angle D. Find the value of Angle B.
19. Two concentric circles have radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
20. In a circle with center O, a chord AB is drawn. A point P on the chord divides it such that AP = 3 cm and PB = 5 cm. If the distance OP is 4 cm, calculate the radius of the circle.

Answer Key

Section A: MCQs

1. C | 2. A | 3. C | 4. A | 5. B | 6. A | 7. B | 8. B | 9. A | 10. B | 11. A | 12. C | 13. D | 14. C | 15. A | 16. B | 17. A | 18. B | 19. B | 20. B

Section B: Subjective

1. 0 cm (The point is outside the circle, so no chord can pass through it)
2. 108 degrees 3. 10 cm 4. 100 degrees 5. 90 degrees
6. 48 cm 7. Rectangle 8. 5 cm 9. 4 cm
10. On the circumference of a single circle (Concyclic) 11. 65 degrees
12. 4 cm 13. square root of 2 : 1 14. 2 cm
15. square root of ((AE - BE) square / 2) 16. 30 degrees
17. AB = AP (They are equal) 18. 150 degrees 19. 8 cm
20. square root of 31 cm

Quantum MCQ

1. A chord of length 16 cm is at a distance of 6 cm from the center of a circle. What is the length of a chord of the same circle which is at a distance of 8 cm from the center?
(A) 12 cm
(B) 14 cm
(C) 10 cm
(D) 16 cm
2. In a circle, chord AB and chord CD intersect at an internal point P. If $AP = 4$ cm, $PB = 6$ cm, and $CP = 3$ cm, find the length of PD.
(A) 5 cm
(B) 8 cm
(C) 7 cm
(D) 9 cm
3. An equilateral triangle ABC is inscribed in a circle of radius r . The distance of the side BC from the center of the circle is:
(A) $r / 2$
(B) $r / \sqrt{3}$
(C) $r\sqrt{3} / 2$
(D) $r / 3$
4. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, then the two chords are:
(A) Perpendicular to each other
(B) Parallel to each other
(C) Equal in length
(D) Of completely random lengths
5. In a cyclic quadrilateral ABCD, if $\text{angle } A - \text{angle } C = 40$ degrees, then the measure of the smaller angle between them is:
(A) 60 degrees
(B) 70 degrees
(C) 80 degrees
(D) 110 degrees
6. Three points A, B, C lie on a circle such that $\text{angle } ABC = 120$ degrees. If the radius of the circle is 6 cm, the length of the minor arc AC is:
(A) $\frac{2}{\pi}$ cm

(B) $\frac{4}{\pi}$ cm

(C) $\frac{6}{\pi}$ cm

(D) $\frac{8}{\pi}$ cm

7. The length of the common chord of two intersecting circles of radii 15 cm and 20 cm, whose centers are 25 cm apart, is:

(A) 12 cm

(B) 18 cm

(C) 24 cm

(D) 30 cm

8. In a circle with center O, AB and CD are two parallel chords on opposite sides of O such that AB = 10 cm, CD = 24 cm. If the distance between AB and CD is 17 cm, the radius of the circle is:

(A) 12 cm

(B) 13 cm

(C) 15 cm

(D) 17 cm

9. If a cyclic trapezium has its non-parallel sides equal, and its diagonals intersect at an angle of 90 degrees, then the trapezium must be:

(A) A scalene trapezium

(B) An isosceles trapezium

(C) A rhombus

(D) A rectangle

10. A, B, C, D are four points on a circle in order. If angle ADB = 45 degrees and angle BAC = 35 degrees, then the measure of angle CBD is:

(A) 35 degrees

(B) 45 degrees

(C) 80 degrees

(D) 100 degrees

11. The radius of the circumcircle of a right-angled triangle whose sides containing the right angle are 6 cm and 8 cm is:

(A) 5 cm

(B) 10 cm

(C) 7 cm

(D) 4 cm

12. In a cyclic quadrilateral ABCD, diagonals AC and BD intersect at E. If angle DBC = 70 degrees and angle BAC = 30 degrees, find angle BCD.

- (A) 70 degrees
- (B) 80 degrees
- (C) 90 degrees
- (D) 100 degrees

13. O is the circumcenter of triangle ABC. If angle OBC = 35 degrees, then the measure of angle BAC is:

- (A) 35 degrees
- (B) 55 degrees
- (C) 65 degrees
- (D) 70 degrees

14. Two circles touch each other internally. The sum of their areas is $116\pi \text{ cm}^2$ and the distance between their centers is 6 cm. The radius of the larger circle is:

- (A) 4 cm
- (B) 6 cm
- (C) 8 cm
- (D) 10 cm

15. If a regular hexagon is inscribed in a circle of radius r , the perimeter of the hexagon is:

- (A) $3r$
- (B) $6r$
- (C) $3\sqrt{3}r$
- (D) $4\sqrt{2}r$

16. In a circle, the major arc is three times the minor arc. The corresponding central angle of the minor arc is:

- (A) 60 degrees
- (B) 90 degrees
- (C) 120 degrees
- (D) 75 degrees

17. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the segment, then the four points are:

- (A) Collinear
- (B) Concyclic
- (C) Vertices of a square
- (D) Vertices of a parallelogram

18. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and angle ADC = 140 degrees, then angle BAC is:

- (A) 40 degrees
- (B) 50 degrees
- (C) 30 degrees
- (D) 60 degrees

19. P is a point inside a circle of radius 5 cm. If the shortest distance from P to the circle is 2 cm, what is the length of the shortest chord passing through P?

- (A) 6 cm
- (B) 8 cm
- (C) 4 cm
- (D) 5 cm

20. The locus of the midpoints of all chords of a circle running parallel to a fixed chord is:

- (A) A straight line passing through the center perpendicular to the chords
- (B) A circle concentric with the given circle
- (C) A line parallel to the given chords
- (D) A single point

Part 2: Subjective Questions

1. Prove that equal chords of a circle subtend equal angles at the center.
2. In a circle of radius r , two parallel chords of length $2a$ and $2b$ are given. Show that the distance between the chords is $\sqrt{r^2 - a^2} \pm \sqrt{r^2 - b^2}$. Explain under what conditions each sign holds.
3. Prove that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
4. If a cyclic quadrilateral ABCD has perpendicular diagonals intersecting at P, prove that the line drawn from P perpendicular to any side bisects the opposite side (Brahmagupta's Theorem).
5. Two circles intersect at two points A and B. Through A, two line segments PAQ and RAS are drawn to intersect the circles at P, Q and R, S respectively. Prove that angle PBR = angle QBS.
6. AB and AC are two equal chords of a circle with center O. Show that AO is the perpendicular bisector of BC.
7. Prove that a cyclic parallelogram is a rectangle.
8. In triangle ABC, the internal bisector of angle A meets the circumcircle of the triangle at D. Prove that $DI = DB = DC$, where I is the incenter of triangle ABC.
9. AB is a diameter of a circle. CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that angle AEB = 60

degrees.

10. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.
11. Prove that the circles described on the equal sides of an isosceles triangle as diameters intersect at the midpoint of the base.
12. In a cyclic quadrilateral ABCD, prove Ptolemy's Theorem: $AC \cdot BD = AB \cdot CD + BC \cdot AD$.
13. O is the circumcenter of a triangle ABC and OD is perpendicular to BC. Prove that $\angle BOD = \angle A$.
14. Two chords AB and CD of a circle intersect at an external point P. Prove that $PA \cdot PB = PC \cdot PD$.
15. If a line intersects two concentric circles (circles with the same center) with center O at A, B, C, and D sequentially, prove that $AB = CD$.
16. Prove that the angle in a segment shorter than a semicircle is greater than a right angle (obtuse), and the angle in a segment greater than a semicircle is less than a right angle (acute).
17. Bisectors of angles A, B, and C of a triangle ABC intersect its circumcircle at D, E, and F respectively. Prove that the angles of the triangle DEF are $90^\circ - \frac{1}{2} \angle A$, $90^\circ - \frac{1}{2} \angle B$, and $90^\circ - \frac{1}{2} \angle C$.
18. Show that the locus of the orthocenter of a triangle, given its base and vertical angle fixed, forms an arc of a circle.
19. ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC at E and F respectively. Prove that EF is parallel to CD.
20. Let P be a point inside a circle with center O. A chord AB passes through P. Prove that the product $PA \cdot PB$ remains constant for any chord passing through P, and evaluate this constant in terms of the circle's radius R and distance $OP = d$.

Answer Key

1. (A) 2. (B) 3. (A) 4. (C) 5. (B) 6. (D) 7. (C) 8. (B) 9. (B) 10. (B) 11. (A)
12. (B) 13. (B) 14. (D) 15. (B) 16. (B) 17. (B) 18. (B) 19. (B) 20. (A)

Quantum Subjective

1. Use triangle OAB congruent to triangle OCD by SSS criterion ($OA=OC$, $OB=OD$ as radii, $AB=CD$ given). Thus $\angle AOB = \angle COD$ by CPCTC.
2. Let distances from center be x and y. Then $x = \sqrt{r^2 - a^2}$ and $y =$

$\sqrt{r^2 - b^2}$. If chords are on opposite sides of the center, distance is $x+y$.
If on the same side, distance is $|x-y|$.

3. Draw the diameter through the point on the remaining arc and split into two isosceles triangles to show the exterior angle is twice the interior angle.
4. Let PM be perpendicular to CD and extend MP to meet AB at X. Trace angles using cyclic properties and right triangles to show triangle PXA and triangle PXB are isosceles, hence $AX=XB$.
5. Connect AB. angle PBR = 180 degrees - angle PAR (cyclic quad property). Show equivalence via supplementary configurations with angle QBS.
6. triangle OAB congruent to triangle OAC by SSS. Hence angle BAO = angle CAO. This makes AO the angle bisector in isosceles triangle ABC, meaning it acts as the perpendicular bisector of the base BC.
7. Opposite angles of a parallelogram are equal (angle A = angle C) and in a cyclic quad they sum to 180 degrees. Thus, 2 angle A = 180 degrees $\rightarrow$ angle A = 90 degrees. A parallelogram with a right angle is a rectangle.
8. Show angle IBD = angle IBD by calculating angles in terms of angle A and angle B. Since triangle DBD is isosceles, the distances match.
9. triangle OCD is equilateral $\rightarrow$ angle COD = 60 degrees $\rightarrow$ angle CBD = 30 degrees. Since AB is a diameter, angle ADB = 90 degrees $\rightarrow$ angle EDB = 90 degrees. In triangle EDB, angle AEB = 180 degrees - 90 degrees - 30 degrees = 60 degrees.
10. Drop perpendiculars from top vertices to the base. Prove congruency of the two right-angled triangles to establish that base angles are equal, which ensures opposite angles sum to 180 degrees.
11. Let D be the intersection point. angle ADB = 90 degrees (angle in a semicircle). In an isosceles triangle, the altitude to the base bisects the base.
12. Proven by constructing a point K on AC such that angle ABK = angle CBD and using similar triangles (triangle ABK ~ triangle DBC and triangle ABD ~ triangle KBC).
13. OD is perpendicular to BC $\rightarrow$ OD bisects angle BOC. Since angle BOC = 2 angle A, half of it (angle BOD) must equal angle A.
14. Prove triangle PAC ~ triangle PDB using AA similarity (angle P is common, angle PCA = angle PBA because they are exterior/interior opposite angles of cyclic quad ABDC). Thus $PA/PD = PC/PB$.
15. Drop a perpendicular OM to AD. OM bisects AD ($AM=MD$) and also bisects

BC ($BM=MC$). Subtracting equations gives $AM - BM = MD - MC \rightarrow AB = CD$.

16. Use the arc-angle theorem: minor arcs contain angles > 90 degrees because the reflex central angle divided by 2 is > 90 degrees.
17. Bisectors of angles A, B, and C of a triangle ABC intersect its circumcircle at D, E, and F respectively. Prove that the angles of the triangle DEF are 90 degrees $- \frac{1}{2}$ angle A, 90 degrees $- \frac{1}{2}$ angle B, and 90 degrees $- \frac{1}{2}$ angle C.
18. Show that angle BHC = 180 degrees $-$ angle A. Since angle A is fixed, angle BHC is constant, describing a circular arc configuration symmetric to the circumcircle.
19. Show exterior angle angle CFE = angle CDE via parallel internal properties and cyclic supplementary conversions.
20. By Power of a Point, $PA * PB = R^2 - d^2$. This is derived by constructing the chord through the center and using the difference of squares via similar right triangles.