

NCERT Class 9 Mathematics (Ganita Manjari)

Chapter 3: The World of Numbers

Section A: Multiple Choice Questions (20 MCQs)

Alpha Level

Choose the correct alternative for each of the following questions:

1. Which of the following is the smallest natural number?
 - a) 0
 - b) 1
 - c) -1
 - d) 2
2. The set of whole numbers includes all natural numbers along with which number?
 - a) -1
 - b) $\sqrt{2}$
 - c) 0
 - d) $\frac{1}{2}$
3. Ancient humans tracked numbers using marks carved on bones. What are these marks called?
 - a) Decimal points
 - b) Tally marks or notches
 - c) Fractions
 - d) Algebraic variables
4. The Lebombo Bone, an ancient mathematical artifact, was discovered in which continent?
 - a) Asia
 - b) Europe
 - c) Africa
 - d) Australia

5. What is the value of any number multiplied by zero?
 - a) The number itself
 - b) 1
 - c) 0
 - d) Infinite
6. Which of the following is an integer but NOT a whole number?
 - a) 5
 - b) 0
 - c) -3
 - d) 7
7. A rational number is written in the form p/q . What is the condition on q ?
 - a) q must be 0
 - b) q must not be equal to 0
 - c) q must be negative
 - d) q must be a prime number
8. Which of the following numbers is a rational number?
 - a) $\sqrt{3}$
 - b) π
 - c) $4/5$
 - d) $\sqrt{7}$
9. What is the additive inverse of 5?
 - a) $1/5$
 - b) -5
 - c) 0
 - d) 1
10. What is the multiplicative inverse (reciprocal) of $2/3$?
 - a) $-2/3$
 - b) $3/2$
 - c) 1
 - d) 0
11. According to the closure property, the sum of any two integers is always:
 - a) A fraction
 - b) An integer
 - c) An irrational number
 - d) Zero
12. What is the value of $8 + 0$?
 - a) 0

- b) 80
 - c) 8
 - d) -8
13. What is the absolute value of -9?
- a) -9
 - b) 9
 - c) 0
 - d) $\frac{1}{9}$
14. The decimal representation of $\frac{1}{2}$ is:
- a) 0.2
 - b) 0.5
 - c) 0.25
 - d) 1.2
15. A decimal that stops after a finite number of digits is called:
- a) Terminating decimal
 - b) Non-terminating decimal
 - c) Repeating decimal
 - d) Irrational decimal
16. Which of the following is a terminating decimal?
- a) $\frac{1}{3}$
 - b) 0.333...
 - c) $\frac{3}{5}$
 - d) π
17. The number 0.7777... can be written as:
- a) 0.7
 - b) $0.\overline{7}$
 - c) 7.7
 - d) 0.07
18. Which of the following values represents a perfect square under a square root, making it a rational number?
- a) $\sqrt{2}$
 - b) $\sqrt{5}$
 - c) $\sqrt{9}$
 - d) $\sqrt{15}$
19. If you plot numbers on a standard horizontal number line, numbers to the right of zero are always:

- a) Negative
 - b) Positive
 - c) Imaginary
 - d) Fractions only
20. Which basic mathematical property states that changing the order of numbers during addition does not change the sum (e.g., $a + b = b + a$)?
- a) Associative Property
 - b) Distributive Property
 - c) Commutative Property
 - d) Closure Property

MCQ Answer Key Alpha Level

Question Range	Answers (One-Line Format)
1 – 20	1-b, 2-c, 3-b, 4-c, 5-c, 6-c, 7-b, 8-c, 9-b, 10-b, 11-b, 12-c, 13-b, 14-b, 15-a, 16-c, 17-b, 18-c, 19-b, 20-c

Section B: Subjective Questions (20 Questions) Alpha Level

Provide simple, clear steps for each answer:

- Define natural numbers and write down the first five natural numbers.
- Explain the difference between a natural number and a whole number.
- What ancient bone with 29 markings is considered one of the earliest pieces of evidence of counting? Write its name.
- State what the symbol "bindu" (dot) stood for in ancient Indian mathematics.
- Calculate the sum of -15 and 7.
- Find the value of $(-4) \times (-6)$.
- Define a rational number in your own words and give two examples.
- Write the number 0 as a rational number in p/q form.
- Find the additive inverse of the following numbers: (a) -8, (b) $3/4$.
- Find the multiplicative inverse of: (a) 7, (b) $-5/9$.

11. Verify that $4 + 5 = 5 + 4$. Which property of addition does this show?
12. Show that $3 \times (2 \times 5) = (3 \times 2) \times 5$ by calculating both sides.
13. Find the absolute value of: (a) -12, (b) 25.
14. Find a single rational number exactly halfway between 2 and 4.
15. Convert the fraction $\frac{3}{10}$ into its decimal form.
16. Convert the fraction $\frac{1}{4}$ into its decimal form. Explain if it is terminating or non-terminating.
17. Classify the following numbers as Whole Numbers, Integers, or Rational Numbers: -5, $\frac{4}{7}$, 9, 0.
18. Identify which of the following roots are rational numbers: $\sqrt{16}$, $\sqrt{3}$, $\sqrt{100}$, $\sqrt{8}$.
19. Draw a simple sketch description showing where the numbers -2, 0, and 3 would be located on a standard horizontal number line.
20. What is the identity element for multiplication? Show what happens when you multiply a number by it.

Section B: Multiple Choice Questions (20 MCQs) Beta Level

Choose the correct alternative for each of the following questions:

21. Which ancient artifact, dating back approximately 35,000 years, contains 29 carved notches indicating the earliest known use of natural numbers?
 - a) Ishango Bone
 - b) Lebombo Bone
 - c) Bakhshali Manuscript
 - d) Harappan Scales
22. The Ishango Bone (~20,000 BCE) contains a column with asymmetric notches grouping 11, 13, 17, and 19. What mathematical concept do these numbers represent?
 - a) Perfect Squares
 - b) Fibonacci Sequence
 - c) Prime Numbers
 - d) Even Numbers
23. Which Indian mathematician formally defined zero as the result of subtracting a

number from itself ($a - a = 0$) in his work *Brāhmasphuṭasiddhānta*?

- a) Aryabhata
- b) Brahmagupta
- c) Bhaskara II
- d) Madhava

24. The early use of a dot (bindu) as a symbol for zero is prominently found in which historical document?

- a) Rigveda
- b) Lalitavistara Sutra
- c) Bakhshali Manuscript
- d) Sulba Sutras

25. If a is a non-zero rational number, what is the value of a multiplied by its multiplicative inverse?

- a) 0
- b) 1
- c) -1
- d) a^2

26. Which of the following properties is NOT always satisfied by the set of rational numbers under the operation of division?

- a) Commutative Property
- b) Associative Property
- c) Closure Property
- d) All of the above

27. What is the absolute value of the expression $|-7 + 3| - |4 - 9|$?

- a) -1
- b) 1
- c) 9
- d) -9

28. How many rational numbers lie strictly between $\frac{3}{5}$ and $\frac{4}{5}$?

- a) Only one
- b) Exactly ten
- c) Infinitely many
- d) Zero

29. Which of the following is a rational number between $\frac{1}{3}$ and $\frac{1}{2}$?

- a) $\frac{5}{12}$
- b) $\frac{2}{9}$
- c) $\frac{7}{12}$

- d) $\frac{4}{15}$
30. When expressed as a decimal, the number π (pi) is:
- a) Terminating
 - b) Non-terminating repeating
 - c) Non-terminating non-repeating
 - d) Finite decimal
31. The decimal representation of the rational number $\frac{7}{8}$ is:
- a) 0.875
 - b) 0.785
 - c) 0.885
 - d) 0.877
32. Which of the following represents the rational number $0.4777\ldots$ (or $0.\overline{47}$) in the form $\frac{p}{q}$?
- a) $\frac{47}{99}$
 - b) $\frac{43}{90}$
 - c) $\frac{43}{99}$
 - d) $\frac{47}{90}$
33. The product of a non-zero rational number and an irrational number is always:
- a) An integer
 - b) A rational number
 - c) An irrational number
 - d) A natural number
34. Which of the following numbers is irrational?
- a) $\sqrt{4}$
 - b) $\sqrt{9}$
 - c) $\sqrt{12}$
 - d) $\sqrt{15}$
35. Between any two distinct real numbers, there exist:
- a) Only rational numbers
 - b) Only irrational numbers
 - c) Infinitely many rational and irrational numbers
 - d) Finite real numbers
36. What is the decimal equivalent of the rational number $\frac{1}{7}$?
- a) 0.142857...
 - b) 0.1444...
 - c) 0.285714...
 - d) 0.15625

37. If x and y are two rational numbers such that $x < y$, then which of the following is guaranteed to be a rational number between them?
- a) $(x + y) / 2$
 - b) $xy / 2$
 - c) $(y - x) / 2$
 - d) $\sqrt{xy}$
38. What is the additive identity for rational numbers?
- a) 1
 - b) -1
 - c) 0
 - d) Does not exist
39. The value of $(-5) \times (-4) \times (-2)$ is equal to:
- a) 40
 - b) -40
 - c) 20
 - d) -20
40. Which of the following numbers has a non-terminating but cyclic (repeating) decimal expansion?
- a) $1/5$
 - b) $3/2$
 - c) $2/3$
 - d) $7/25$

Section B: Subjective Questions (20 Questions) Beta Level

Provide detailed, step-by-step solutions for the following problems:

- 21. Briefly explain the historical significance of the Lebombo Bone and the Ishango Bone in the evolution of numerical tracking.
- 22. State the rules formulated by Brahmagupta for arithmetic operations involving zero (Addition, Subtraction, and Multiplication).
- 23. Verify the commutative property of addition for the rational numbers $-3/4$ and $5/6$.
- 24. Show that the associative property does not hold for the subtraction of rational numbers using an example with $1/2$, $1/3$, and $1/4$.
- 25. Find five rational numbers between $2/5$ and $3/4$ by equating their denominators.
- 26. Find three rational numbers lying strictly between $-1/3$ and $1/2$.

27. Represent the rational numbers $\frac{3}{4}$, $-\frac{5}{4}$, and $\frac{7}{4}$ on a single cleanly drawn number line.
28. Express the repeating decimal $0.6666\ldots$ ($0.\overline{6}$) in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
29. Convert the pure recurring decimal $0.232323\ldots$ ($0.\overline{23}$) into a fractional form in its simplest terms.
30. Express the mixed recurring decimal $0.123333\ldots$ ($0.12\overline{3}$) in the form $\frac{p}{q}$.
31. Prove by contradiction that $\sqrt{2}$ is an irrational number. (Assume $\sqrt{2} = \frac{p}{q}$ in simplest form and derive a contradiction).
32. Describe the geometric step-by-step method to locate the position of $\sqrt{2}$ on the number line using a right-angled triangle.
33. Examine whether the number $(3 + \sqrt{23}) - \sqrt{23}$ is rational or irrational. Simplify completely to justify your answer.
34. Simplify the expression $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$ using algebraic identities and state whether the result is rational or irrational.
35. Classify the following numbers as rational or irrational, giving a brief reason for each: (a) $\sqrt{225}$, (b) $\sqrt{7}$, (c) 0.3796 , (d) $7.478478\ldots$
36. Evaluate the expression: $(-12) \times (-5) + (-18) \div 3 - |-15|$. Show each operational step clearly.
37. Define absolute value. If $x = -\frac{5}{6}$ and $y = \frac{2}{3}$, evaluate $|x + y|$ and compare it with $|x| + |y|$. What inequality does this illustrate?
38. Explain the concept of "One-to-One Correspondence" in ancient counting systems, using the historical context of counting cattle with pebbles.
39. If a rational number has a terminating decimal expansion, what condition must be satisfied by the prime factorization of its denominator when the fraction is in its simplest form?
40. Find two irrational numbers lying between the rational numbers 0.12 and 0.13 .

Section C: MCQ Answer Key

Question Range	Answers (One-Line Format)
1 – 20	1-b, 2-c, 3-b, 4-c, 5-b, 6-d, 7-a, 8-c, 9-a, 10-c, 11-a, 12-b, 13-c, 14-c, 15-c, 16-a, 17-a, 18-c, 19-b, 20-c

Section A: Multiple Choice Questions (20 MCQs)

Delta Level

Choose the correct alternative for each of the following questions:

41. Consider the set of all irrational numbers. Under which of the following standard operations is this set closed?
 - a) Addition
 - b) Multiplication
 - c) Division
 - d) None of the above
42. If p is a prime number, which of the following statements concerning $\sqrt{p}$ is logically equivalent to its irrationality?
 - a) Its decimal expansion has no repeating block of finite length.
 - b) It cannot be written as the root of any polynomial equation.
 - c) The square of the number is always a composite integer.
 - d) It is greater than any integer.
43. Let $x = 0.12112111211112\dots$ and $y = 0.13113111311113\dots$. Which of the following is true about the sum $x + y$?
 - a) It is definitely rational.
 - b) It is definitely irrational.
 - c) It is rational only if $x = y$.
 - d) It cannot be determined without terminating forms.
44. Which property of real numbers ensures that every non-empty set of real numbers that is bounded above has a least upper bound?
 - a) Archimedean Property
 - b) Completeness Property
 - c) Density Property
 - d) Commutative Property
45. The absolute value function satisfies the triangle inequality $|a + b| \leq |a| + |b|$. Under what exact condition does $|a + b| = |a| + |b|$ hold for non-zero real numbers?
 - a) a and b have opposite signs.
 - b) a and b have the same sign.
 - c) a must be greater than b .

- d) Either a or b must be an integer.
46. If the rational number p/q (in lowest terms) has a terminating decimal expansion, what must be the form of the prime factorization of q ?
- $3^m \times 5^n$
 - $2^m \times 5^n$ (where $m, n \geq 0$)
 - $2^m \times 3^n$
 - Any prime factor other than 7.
47. What is the length of the maximum possible repeating block of digits (period length) in the decimal expansion of the rational fraction $1/19$?
- 19
 - 18
 - 9
 - 20
48. If n is a positive integer that is not a perfect square, then $\sqrt{n}$ is:
- A rational number
 - An irrational number
 - An integer
 - A natural number
49. Express the mixed recurring decimal $2.3454545\dots$ (or $2.34\overline{5}$) as a rational fraction in its lowest terms. What is its denominator?
- 990
 - 495
 - 110
 - 999
50. Which of the following numbers is transcendently irrational (cannot be a root of a non-zero polynomial with rational coefficients)?
- $\sqrt{3} + \sqrt{2}$
 - π
 - $\sqrt{5}$
 - $0.\overline{3}$
51. Between any two distinct rational numbers r_1 and r_2 , how many irrational numbers exist?
- None
 - Exactly one
 - A finite number
 - Infinitely many
52. In the historical context of the *Bakhshali Manuscript*, the usage of the "bindu" as a

placeholder represents the birth of what structural concept?

- a) Positional Place-Value System
- b) Geometric Progression
- c) Irrational Number Proofs
- d) Negative Fractions

53. The value of $\sqrt{11 + 6\sqrt{2}}$ is equal to which of the following simplified expressions?

- a) $3 + \sqrt{2}$
- b) $3 - \sqrt{2}$
- c) $2 + \sqrt{3}$
- d) $1 + 3\sqrt{2}$

54. What can be said about the number 0.101001000100001...?

- a) It is a rational number with non-periodic expansion.
- b) It is an irrational number because it is non-terminating and non-recurring.
- c) It can be expressed as a fraction p/q where q is a power of 10.
- d) It is a repeating decimal with period 1.

55. If a is an irrational number, then $1/a$ is:

- a) Always rational
- b) Always irrational
- c) Sometime rational, sometimes irrational
- d) An integer

56. What is the value of the expression $|-x|$ if $x < 0$?

- a) x
- b) $-x$
- c) 0
- d) $1/x$

57. Which ancient Indian text contains geometric dynamic methods for approximating the square root of 2 up to five decimal places using fractional series?

- a) Aryabhatiya
- b) Sulba Sutras
- c) Lilavati
- d) Khandakhadyaka

58. If a, b, c are integers and $a > b$, under what conditions is $ac < bc$ always guaranteed?

- a) $c > 0$
- b) $c < 0$
- c) $c = 0$

- d) For any value of c
59. The sum of two numbers, one rational and one irrational, is:
- Rational
 - Irrational
 - An integer
 - Zero
60. What is the fractional simplest form of the recurring sequence $0.99999\dots$?
- $99/100$
 - $9/10$
 - 1
 - It never reaches an integer value.

MCQ Answer Key

Question Range	Answers (One-Line Format)
1 – 20	1-d, 2-a, 3-b, 4-b, 5-b, 6-b, 7-b, 8-b, 9-b, 10-b, 11-d, 12-a, 13-a, 14-b, 15-b, 16-b, 17-b, 18-b, 19-b, 20-c

Section B: Subjective Questions (20 Questions) Delta Level

Provide complete, rigorous proofs and explicit operational mathematical reasoning for the following:

- Prove by counterexample that the set of irrational numbers is not closed under subtraction or division. Give clear numerical examples.
- Let a be a non-zero rational number and b be an irrational number. Formally prove that the product ab is always an irrational number using a proof by contradiction.
- Find three distinct irrational numbers lying strictly between the fractions $5/7$ and $9/11$. Show your complete decimal derivation methodology.
- Convert the complex recurring decimal $4.21353535\dots$ ($4.21\overline{35}$) into its equivalent standard rational fraction p/q in its most reduced form.
- Rigorously prove that $\sqrt{3}$ is an irrational number using the method of

contradiction and prime factors.

46. Provide a formal algebraic proof of the dynamic triangle inequality $|x - y| \geq ||x| - |y||$ for all real numbers x and y .
47. Using a geometric construction paradigm based on Pythagoras' Theorem, describe how you would successively locate $\sqrt{5}$, $\sqrt{6}$, and $\sqrt{7}$ on a continuous real number line.
48. Analyze the prime factor layout of the denominator of $1/13$. Determine the maximum decimal cyclic length without executing the complete long division, and explain the underlying arithmetic rule.
49. Determine whether the number $(\sqrt{2} + \sqrt{3})^2$ is rational or irrational. Expand the terms step by step to validate your final stance.
50. Rationalize the denominator of the advanced expression: $1 / (\sqrt{2} + \sqrt{3} - \sqrt{5})$. Show all intermediate rationalization steps clearly.
51. If x is a rational number and y is an irrational number, under what precise mathematical constraint will their sum $x + y$ turn into a rational integer? Prove your answer.
52. Explain the mathematical synthesis behind Brahmagupta's rules regarding operations with "Khakhila" (zero), specifically evaluating why he encountered systemic issues when defining division by zero.
53. Examine the decimal representation of $1/7 = 0.142857$. Without long division, find the exact decimal expansion of $3/7$, $4/7$, and $6/7$ using cyclic shift logic.
54. Find the value of the algebraic expression $\sqrt{(7 - 4\sqrt{3})}$ by converting the radicand into a perfect squared binomial framework.
55. Let a, b, c be real numbers. If $a < b$ and $b < c$, prove that $a < c$. Name the axiom property governing this relationship.
56. If $x = 0.99999\dots$, prove algebraically that x is exactly equal to 1. Address why this conclusion is philosophically and mathematically sound.
57. Discuss the historical transition from "one-to-one tally matching" (like the Ishango bone notches) to abstract symbolic representations of numbers within early civilizations mentioned in *Ganita Manjari*.
58. Show that between any two distinct real numbers, there are infinitely many rational numbers using the density property of real numbers.
59. If $x = 3 - 2\sqrt{2}$, evaluate the exact value of the expression $x^2 + 1/x^2$.
60. Prove that if a is a rational number and b is an integer, then the expression $(a^2 - b^2)$ is always a rational number.

Section A: Multiple Choice Questions (20 MCQs)

Quantum Level

Choose the single correct alternative for each problem:

61. Let S be the set of natural numbers that can be grouped into prime number clusters mimicking the asymmetric notches found on the Ishango bone. If a number N is the product of four distinct primes sequence (p_1, p_2, p_3, p_4) which are consecutive primes starting from 11, how many distinct factors does N^2 have?
- a) 16
 - b) 81
 - c) 256
 - d) 625
62. An ancient placeholder system uses a dot (bindu) to represent missing places. If a five-digit number in base 10 is written as $7 \cdot 2 \cdot 5$, where the dots represent unknown identical single digits d , for how many distinct values of d is this number divisible by 9?
- a) 1
 - b) 2
 - c) 0
 - d) 9
63. If x and y are distinct non-zero rational numbers such that $x + y$ and $x - y$ are both integers, then which of the following statements must always be true?
- a) Both x and y are integers.
 - b) $2x$ and $2y$ are both integers.
 - c) The product xy is an integer.
 - d) The fraction x/y is an integer.
64. Let the fractional period length of a prime p in its decimal representation be denoted by $\lambda(p)$. If the decimal expansion of $1/p$ has a pure repeating cyclic block of exactly 6 digits, what is the maximum possible value of the prime p ?
- a) 7
 - b) 13
 - c) 37
 - d) 97
65. If the expression $|x - 2| + |x - 5| + |x - 9|$ achieves its absolute minimum value for a real number x , what is that unique minimum value?

- a) 7
- b) 9
- c) 4
- d) 11

66. Let a, b, c be distinct positive rational numbers such that the expression

$\sqrt{a} + \sqrt{b} = \sqrt{c}$ Which of the following statements must be true?

- a) Each individual term $\sqrt{a}$, $\sqrt{b}$, and $\sqrt{c}$ must be rational.
- b) The product ab must be a perfect square of a rational number.
- c) The sum $a + b$ must equal c .
- d) None of the above are necessarily true.

67. The pure recurring decimal $0.ababab\dots$ (where a and b are distinct digits from 0 to 9) is converted into its simplified rational form p/q . If q is a two-digit prime number, how many pairs of (a, b) exist?

- a) 8
- b) 9
- c) 10
- d) 18

68. Let k be an integer. For how many integer values of k is the expression $(3k + 4) / (k - 2)$ itself a terminating integer?

- a) 2
- b) 4
- c) 8
- d) Infinitely many

69. If n is a natural number such that $1 \leq n \leq 100$, for how many values of n will the expression $\sqrt{4n + 1}$ result in a rational number?

- a) 5
- b) 7
- c) 9
- d) 10

70. Consider three real numbers a, b, c satisfying $|a - b| = 3$, $|b - c| = 4$, and $|c - a| = k$. What is the sum of all possible real values of k ?

- a) 7
- b) 8
- c) 1
- d) 14

71. If the decimal expansion of a rational number m/n (in lowest terms) terminates precisely after 4 decimal places, which of the following could be the value of n ?

- a) 250
- b) 625
- c) 1600
- d) 2000

72. Let x be an irrational number. If $x^2 + 4x$ is rational and $x^3 - 20x$ is rational, what is the value of $x^2 + 4x$?

- a) 5
- b) 4
- c) 2
- d) 1

73. How many rational pairs (x, y) satisfy the radical equation $x\sqrt{2} + y\sqrt{3} = \sqrt{5} + 2\sqrt{6}$?

- a) 0
- b) 1
- c) 2
- d) Infinitely many

74. What is the exact fractional value of the infinite nested radical expression

$$\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} ?$$

- a) 2
- b) 3
- c) $\sqrt{6}$
- d) Rational value does not exist.

75. If a and b are rational numbers such that $a + b\sqrt{5} = \frac{1}{(3 - \sqrt{5})}$, what is the value of $a + b$?

- a) $1/2$
- b) 1
- c) $3/4$
- d) 2

76. Let x be a real number such that $x = 0.1010010001\dots$ where the number of zeros increases by one each time. Which of the following is true?

- a) x is rational because it follows a strict predictable rule.
- b) x is rational because it contains only 0 and 1.
- c) x is irrational because its decimal expansion is non-terminating and non-periodic.
- d) x can be expressed as a fraction with denominator 99.

77. The absolute difference between two rational numbers p/q and r/s is less than $1/(qs)$. Which statement must be true?

- a) $p/q = r/s$
 - b) $p = r$ and $q = s$
 - c) The numbers must be consecutive integers.
 - d) No such distinct rational fractions exist.
78. If n is a natural number, what is the highest common factor of the numbers $(n^2 + n)$ and $(n + 1)$?
- a) 1
 - b) n
 - c) $n + 1$
 - d) Dependent on the value of n .
79. If we define a special operation $\oplus$ such that $a \oplus b = |a - b| / (1 + |ab|)$, which of the following properties fails for the set of positive rational numbers?
- a) Closure Property
 - b) Commutative Property
 - c) Associative Property
 - d) None of the above
80. Let x be a positive real number. If the fractional part of x (denoted by $x - \lfloor x \rfloor$) and the number x satisfy the equation $x^2 - 4\lfloor x \rfloor = 3$, how many solutions exist for x ?
- a) 0
 - b) 1
 - c) 2
 - d) 3

MCQ Answer Key

Question Range	Answers (One-Line Format)
1 – 20	1-b, 2-b, 3-b, 4-b, 5-a, 6-b, 7-a, 8-c, 9-b, 10-d, 11-b, 12-b, 13-b, 14-b, 15-b, 16-c, 17-a, 18-d, 19-c, 20-b

Section B: Subjective Questions (20 Olympiad-Level Problems) Quantum Level

Provide exhaustive mathematical proofs, derivations, and systematic rationalizing steps

for each challenge:

61. Let x be a rational number such that $x = p/q$ in its lowest terms. If $1/x + x = 3$, prove rigorously using parity (even/odd properties of integers) why no such rational number x can actually exist.
62. Prove that if $\sqrt{a} + \sqrt{b}$ is a rational number for positive integers a, b, c , then each individual value $\sqrt{a}, \sqrt{b}$, and $\sqrt{c}$ must independently be an integer or an irrational number whose prime components match.
63. Find all pairs of integers (m, n) that satisfy the algebraic constraint: $mn - 3m - 2n = 14$. Show your full factorization breakdown.
64. Without executing the infinite division, prove that the decimal sequence for $1/17$ yields a cyclic period of exactly 16 repeating digits, and show why the sum of the first 8 digits and the last 8 digits of this period always equals a string of 9s.
65. An irrational number x has the property that both $x^2 + 2x$ and $x^3 - 7x$ are rational numbers. Determine the exact value of x and prove its irrationality using the standard Class 9 contradiction workflow.
66. Find the value of the expression $\sqrt{(12 - 4\sqrt{5} - 4\sqrt{3} + 2\sqrt{15})}$ by expressing the terms under the radical as a perfect square of a trinomial expression $(a + b + c)^2$.
67. Let a, b, c be real numbers such that $a + b + c = 0$. Prove that the expression $|a| + |b| \geq |c|$ is always valid, and determine the exact condition required for the expression to become an equality.
68. Rigorously prove that the number $\sqrt{2}, \sqrt{3}, \sqrt{5}$ is irrational. You must use successive squaring and algebraic isolation without referencing higher-grade field theory.
69. A rational fraction p/q has a repeating decimal sequence of $0.\overline{xyz}$. If the value of this fraction is exactly equal to $1 / (x + y + z)$, find all possible distinct values for the three-digit integer 'xyz'.
70. Rationalize the complex denominator of the fraction: $4 / (\sqrt{2} + \sqrt{3} + \sqrt{7})$. Ensure no radical expressions remain in the denominator.
71. Let x and y be positive real numbers. If $x + y = 1$, prove that $(1 + 1/x)(1 + 1/y) \geq 9$ using purely basic Class 9 algebraic expansions and inequalities.
72. Find two rational numbers between $\sqrt{2}$ and $\sqrt{3}$ that have a terminating decimal representation containing exactly two non-zero decimal digits. Show your structural bounding methodology.
73. Prove that for any natural number n , the expression $\sqrt{n^2 + 3n + 1}$ can never represent a rational integer.
74. Simplify completely into a form free of nested radicals: $\sqrt{(6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6})}$.
75. Let a, b, c be integers such that $a + b\sqrt{2} + c\sqrt{3} = 0$. Prove that $a = b = c = 0$.

76. Find all real numbers x that satisfy the multi-absolute value equation: $|x - 1| + |x - 3| = |x - 4|$. Provide clean interval analysis testing.
77. Let a and b be rational numbers. If the value of $(a^2 - 2b^2)^2 + 8ab(a^2 - 2b^2) = 0$, find all possible rational solutions for a and b .
78. Express the repeating decimal $0.1123123123\dots$ ($0.1123\overline{}$) into a fraction p/q in its simplest form, and find the prime factorization of the resulting denominator q .
79. If $x = (\sqrt{5} + 1) / 2$, find the exact value of the algebraic expression $x^5 - 3x^4 + 2x^3 - x^2 - x + 7$ without calculating the fifth power directly. (Hint: look for a lower-degree polynomial relationship).
80. Prove that the sum of the cubes of any three consecutive natural numbers is always divisible by 3, and connect this to the properties of integers discussed in the historical sections of the chapter.